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CONSTANTS OF LUNAR PHYSICAL LIBRATION

by A. A. Gorynya

"Naukova Dumka" Press, Kiev, 1969

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CONSTANTS OF LUNAR PHYSICAL LIBRATION

By A. A. Gorynya

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ANNOTATION

This monograph gives the results of studies of the constants of lunar physical libration on the basis of visual and photographic observations of pairs of craters by the position angles method.

This material is of interest to astronomers and students and graduate students studying lunar figure and rotation.

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CONSTANTS OF LUNAR PHYSICAL LIBRATION

A. A. Gorynya

ABSTRACT. This study gives a thorough review of earlier observations of lunar physical libration and a critique and evaluation of these observations. The author describes in great detail the method which he has employed in determining lunar physical libration. This is the position angles method. It is important because it is the first attempt to dispense with total reliance on the lunar limb zone in determining the parameters of lunar rotation. All the observational data are given in the text and a lengthy Appendix. These values of the constants of lunar rotation must be considered preliminary since they were obtained for an initial value $f_0 = 0.85$. The author is reprocessing the observations for an initial value $f_0 = 0.60$ and the results of this processing will be published in the immediate future.

INTRODUCTION

One of the most complex problems in celestial mechanics is the investigation of the figure and motion of the moon. This problem is being studied jointly by astronomy, geodesy and celestial mechanics. The problems involved in motion, rotation, figure and internal structure of the moon are inter-related.

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Only a precise study of lunar motion makes it possible to compile tables which very rigorously correspond to true lunar motion. This involves great difficulties, one of which is solving the problem of motion of three mutually attracting bodies. In its solution various hypotheses are advanced concerning the distribution of density in a celestial body and its figure. For example, the presence of physical libration indicates that the lunar figure differs noticeably from spherical. Although this asphericity is slight it must be taken into account. The fact that the earth is not strictly spherical makes lunar motion about the earth very complex as well; accordingly, study of lunar motion is a difficult problem in celestial mechanics.

*Numbers in the margin indicate pagination in the foreign text.

Study of lunar rotation about its axis is closely related to its figure; it is characterized by the moments of inertia of the lunar body. It is known that the moon is noticeably distended in an earthward direction. Therefore, small pendulum-like lunar oscillations, called physical libration, are superposed on the optical libration.

Forced physical libration arises and is maintained by the deviation of this distention from a precise earthward direction due to optical libration. The magnitude of physical libration is dependent on the degree of lunar distention, or on the mechanical ellipticity of the moon, which is characterized by the parameter

$$f = \frac{B(C-B)}{A(C-A)},$$

/4

where A, B and C are the main lunar moments of inertia.

The amplitude of this oscillation, visible from the earth, is only about 0".5. As a result of the small value of this oscillation, determination of the parameter f is a subtle and complex problem.

Many studies have been devoted to solution of this problem over a period of more than 150 years. However, they have all been based on observations of the visible lunar limb, which exerts an important effect on positional lunar observations, and accordingly on the derivation of its rotational parameters. Accordingly, allowance for the irregularities of the visible lunar limb is a fundamentally important problem. Maps of the lunar limb profile have been compiled for this purpose at different observatories (Hayn, Przybyllok, Weimer, Nefed'yev and Watts). The compilation of these maps involves great difficulties. Investigation and allowance for irregularities of the lunar limb is of great importance for studying lunar rotation and determining the parameters of lunar rotation.

Despite the great number of studies devoted to determination of the parameters of lunar rotation, there is still no unanimous opinion concerning their true values. The results of data processing reveal that the parameter f has two values. One of them is less than critical (0.662), some investigators considering it to be equal to 0.60; the other value is greater than

critical, 0.73. Various determinations are grouped around these two f values. Accordingly, there is even discussion as to near which of the two mentioned values the true f value is situated. We feel that as long as observations are made by the old method (heliometric, photographic, etc.), it will scarcely be possible to obtain new, more reliable results. For the time being the old observational methods must be discarded and new ones must be sought.

A. A. Yakovkin proposed measurements either of the position angles of craters situated within the apparent lunar disk (in the limb zone) relative to the crater Moesting A, or measurement of the distance from Moesting A to the same craters. In accordance with this proposal, beginning in 1962, the Main Astronomical Observatory of the Academy of Sciences Ukrainian SSR made visual observations of the position angles of the direction between Moesting A and a limb crater using an AVR-2 instrument having a hair eyepiece micrometer with a position circle. This paper gives the results of processing of observations made during the period 1962 through 1966. /5

We selected the visual method because we consider it more precise than the photographic method. However, interesting material can be obtained by using the position angles method for processing lunar photographs taken at the Astronomical Observatory of Kiev State University from 1950 through 1955 (87 photographs).

The basis for our study is the processing of observational materials (a total of 421 evenings of lunar observations) obtained during the period 1950 through 1966 by the new method. In comparison with the classical method of heliometric measurements, employed since the time of Bessel, the position angles method has an indisputable advantage, that is, determination of the parameters of physical libration is not dependent on relief of the limb zone.

This study sets forth the theory of lunar physical libration as given by Hayn, the substance of the position angles and distances method, processing of observations by this method, and use of these data in determining the parameters of lunar rotation (I and f); it also gives a review of studies of physical libration and investigation of the lunar profile and figure.

CHAPTER I

INVESTIGATION OF THE LUNAR PROFILE AND FIGURE

The moon as the earth's natural satellite is an interesting object for investigation, both from the point of view of celestial mechanics and from the astrometric point of view. More work and efforts of experienced observers and outstanding mathematicians have been devoted to the study of lunar motion and rotation than to the study of any other celestial body. It has served as an experimental object for checking the theory of gravitation, because due to its closeness to the earth one can detect the slightest inaccuracy in the law of gravitation or in the conclusions drawn in the theory of perturbations.

Rapid motion and a large parallax make it possible to use the moon for solving astronomical and geodetic problems. Astrometric observations of the moon involve determinations of its coordinates; these are necessary for a more precise determination of its orbit, determining ephemeris time, constructing a fundamental coordinate system in the celestial sphere, etc. Geodetic observations can be used for determining the coordinates of points on the earth's surface, determining distances between individual places, and determining corrections to the earth's size and figure. /6

At the present time two terrestrial ellipsoids are being used in geodetic measurements: one for Europe and Asia, and another for America. The coupling of these two ellipsoids is a difficult but extremely necessary problem.

Study of lunar orbital motion and rotation is closely associated with studies of its figure and profile. The successes in these investigations have been discussed in detail by A. A. Nefed'yev in [27] and therefore we will discuss them here only briefly and add only a review of studies which have been published since 1958.

Lunar Spherical Model

In order to determine the lunar orbital elements it is necessary that the geocentric spherical coordinates of the lunar center of mass be determined from observations. In actual practice they are determined from observations

of the lunar limb. Reduction to the center of mass requires a knowledge of the space selenocentric coordinates of observed points on the lunar limb, i.e., the relief of the lunar limb zone with coordinate origin at the center of mass. The construction of a map showing its image in isohypses is a complex problem. Study of lunar relief is of great importance for astronomy and geodesy. The solution of a number of problems involved in the study of the figure and motion of the moon, problems in selenodesy, and other problems in astrometry and geodesy is dependent on a proper solution of this problem.

This problem arises in every case when it is necessary to process heliometric, photographic and meridian observations, and occultations of stars by the moon, and also when comparing theory with observations.

Study of lunar relief is closely associated with investigations of its total surface. It has great cosmogonic importance and is necessary for drawing conclusions concerning physical conditions on the moon. Irregularities of the lunar limb reach 4-5" and are continuously changing with respect to longitude and latitude as related to optical libration. /7

Although the lunar relief has been studied for a long time, there is still no final solution for this problem. Originally the determinations of lunar limb irregularity were made with a low accuracy. Astronomers now have an enormous amount of observational material (photographic, heliometric, occultations of stars by the moon), adequate for successful solution of this problem.

The lunar profile differs sharply from a circle due to sinuosity and bulges. It is essential to take these irregularities into account because they introduce errors into the results of processing of observations.

The following assumptions have been made in processing heliometric and photographic observations of the crater Moesting A: 1) the lunar profile (neglecting minor relief of a random nature) is a circle, 2) the radius of the adopted circle, adjusted to the mean lunar distance, is a constant value and is not dependent on libration, 3) the centers of the most probable circles coincide with one another for different librations.

The adoption of such assumptions was probably justified by the fact that deviations from them are small and during observations these deviations cannot be detected: the center of the most probable circle passing through 7-11 limb points (in heliometric observations), situated uniformly along an arc of 180° , was used as the center of the lunar figure. In photographic observations the number of selected limb points can be as great as desired, but the method remains the same. It is now well known that the "lunar center" determined in this way does not coincide with the center of mass. This is demonstrated by the fact that the mean correction for lunar longitude experiences a "jump" of $1''.2$ with transition through the time of the full moon [84], whereas the correction of tabulated lunar latitude is always negative. This indicates a movement of the center of the lunar disc relative to the projection of the center of mass onto the picture plane. The correction for tabulated lunar latitude in actuality can vary in different orbital segments from -0.4 to $-0''.2$. It is also dependent on the orbital elements used and on the lunar model employed, taken as the basis for computations (a sphere, as used by Hayn, ellipsoid, etc.). These assumptions, sometimes made even at the present time, are erroneous. The need for introducing a latitude correction is obvious from changes in the figure of the lunar profile in correlation with optical libration. At the southern limb of the moon, in the south polar region, there is an extensive highland which occupies several tens of degrees along the limb arc. Its altitude varies periodically with a change in libration in latitude; this causes corresponding movements of the center of the most probable circle relative to the center of mass with a simultaneous variation in the circle radius. /8

Hansen and Newcomb did not know of this phenomenon and did not take it into account in determining the lunar orbital elements. Meridian observations, used as the basis for computing the lunar orbital elements by Hansen, and later by Newcomb, were processed on the assumption that the four lunar radii, two along the circle of declination to the north and south, and two in a perpendicular direction, are equal to one another and do not vary with libration. Observations, in the case of such processing, in actuality gave the coordinates of a point moving (floating) in the lunar body; this point was assumed to be the

center of mass. If one does not take into account the so-called libration effect, errors are possible when determining the inclination of the lunar orbit. The inclination accepted at the present time must be reduced by $0.3-0''.4$. If observations based on computations of orbital elements are re-processed with allowance for the libration effects, the lunar orbit eccentricity also changes.

What has been said above can naturally be attributed to imperfection in processing observations. In actuality, the systematic deviation of the center of the lunar figure in latitude from its theoretical trajectory, as pointed out by A. A. Yakovkin in [44], is unquestionable; nevertheless, until the present time many researchers have drawn no conclusions from this fact. However, this must be taken into account because all lunar observations (meridian, photographic, heliometric, etc., and therefore selenographic studies, investigation of lunar physical libration) are dependent on a tie-in with points on the lunar limb, and therefore on its assumed ideal configuration. Rigorous computation methods are employed when processing observations, but the processing itself is done on the assumption of a constancy of linear radii of apparent lunar profiles during different librations and the coincidence of the centers of the most probable circles with the center of mass. /9

I. V. Bel'kovich [2], in processing extensive heliometric series of observations, was the first to discover that the radius of the eastern lunar limb is greater than the radius of the western limb by $0''.14$ in the case of average optical libration in latitude. At the present time there is no unanimous opinion concerning the overall figure of the lunar profile: some researchers assume it to be a circle and take into account only minor relief. However, minor relief is only a part of the large systematic deviation of the lunar profile from an optimum circle extending for tens of degrees along the limb.

The first detailed investigation of the lunar profile was made early in the twentieth century by the German Astronomer Hayn. In determining the parameters of lunar physical libration, in 1903 he used a refractor in making a series of micrometric measurements [69] of the distances s between the crater Moesting A and limb points, and also determined their position angles p .

From the conditional equations

$$s - s' = dR' - \cos \delta \sin p d(\alpha_1 - \alpha) - \cos p d(\delta_1 - \delta)$$

Hayn obtained corrections to the assumed value of the lunar radius and determined the coordinates of the crater Moesting A relative to the lunar center $d(\alpha_1 - \alpha)$ and $d(\delta_1 - \delta)$. After analyzing these data, Hayn became convinced that allowance for limb nonuniformity considerably increases the accuracy of the results. In 1907 he proceeded to construct maps of the limb zone, using heliometric observations by Hartwig (1877-1879) and his own photographic observations (1898-1900) for this purpose. Thus, he produced a topographic map of the lunar limb with isohypses drawn each 0".2 (approximately 372 m). Continuing his work, during the years 1908 through 1912 Hayn used the refractor at Leipzig Observatory in obtaining 100 lunar photographs ($F = 3.6$ m, diameter of the lunar image 3.3 cm). A pair of stars was printed on each plate for determining the plate scale and orienting the plates. The lunar photographs were processed in the following way. A mark (prick) was plotted near the apparent center of the lunar image. A coordinate-measuring instrument was used to measure off 45 distances s from the mark to points on the apparent lunar limb. The position angles p for these observations were calculated at the same time. The conditional equations written were used in determining the coordinates for the center of the most probable circle and the correction to the assumed lunar value R_0 . Hayn found the altitudes of 10,500 points on the lunar limb in this way. The zero level for reckoning the altitudes of observed points on the limb on each photograph was its own circle. The limb spherical coordinates P and D were computed for all points. /10

In order to find the lunar limb profile, all the altitudes were determined along the normal line of the limb plotted on the map and were then plotted on the profile. In this procedure an allowance was made for the perspective effect of curvature caused by the fact that the apparent profile is affected in part by the relief of places situated in front of and behind the normal limb line. These maps were published in 1914. A fundamental shortcoming of these maps is that in different regions Hayn gives the altitudes above most probable circles of different radii with noncoinciding centers and that the lunar photographs are not tied in with one another.

In 1908 Przybyllok published corrections to the lunar profile and limb zone maps [8] based on materials from 641 lunar star occultations obtained by Battermann during 1894-1897 using the refractor at the Observatory of the Berlin Academy of Sciences [61]. Battermann's observations were uniformly distributed along the lunar orbit. The position angles of the occultations and reappearances were determined from the lunar radius and the difference in the declinations of its center and the star. If the declination differences exceeded $10'$, the position angles were computed from the differences α and δ of the lunar center and the star. Using Encke's tables, the apparent α' and δ' values were transformed into λ and β ; these were used in computing lunar optical libration.

The position angle of the lunar north pole relative to the equator was computed using auxiliary tables, from which P and D were determined.

Using observation of lunar star occultations, Battermann determined corrections to lunar longitude, some perturbation terms, and the lunar radius. Assuming that this material was inadequate for constructing profiles, Przybyllok used 288 heliometric observations made by Hartwig and 267 observations made by Hayn. Using 1,032 reliable observations of lunar star occultations and heliometric observations, he defined the irregularities of the lunar limb; he illustrated these in his study in the form of tables and maps.

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An important shortcoming in constructing maps on the basis of occultation observations is that occultations are virtually unobservable near the poles. Accordingly, the polar regions remain unstudied. In addition, as pointed out by Rybka in [89], short-period fluctuations in lunar motion can exert an influence on limb irregularity. However, lunar star occultations must without fail be observed. Using these observations it is possible to obtain the baricentric irregularities of the lunar limb, information which can be of great value in lunar studies [18]. Occultation observations yield important material for determining lunar longitude and latitude corrections and for an ephemeridal time service. Their value is considerably increased when registering the times of occultation by the photoelectric method.

For the most part, observations of lunar star occultations have been made and continue to be made by the visual method; accordingly, their accuracy is low (about 0.1 sec). With photoelectric registration times of occultations and reappearances can be observed with an accuracy to 0.001 sec, i.e., two orders of magnitude greater. In addition, the observer's error is completely excluded [3]. However, at the present time the photoelectric method is rarely used. Several such studies have been made in Japan [87] and in America [86]. In Japan regular observations are being made of star occultations by the photoelectric method at the present time. However, these observations can obviously be made only on the dark limb.

In the Soviet Union an attempt has been made to carry out such observations at the Poltava Gravimetric Observatory; unfortunately, this work has been suspended. During the years 1963-1964, V. G. Burachek, using an AVR-2 instrument [3], made test observations of occultations of stars from 5.3 to 6^m.3 by the photoelectric method.

Photographs of the full moon, not having any significant gibbousness (less than 1-2"), obtained one hour before or one hour after a total lunar eclipse, constitute very valuable material for investigating the lunar figure. Under these conditions optical libration in latitude is always close to 0°. The slight gibbousness due to phase is easy to take into account, but then we do not obtain photographs for all other librations. The same shortcoming is characteristic of lunar photographs at the time of an annular solar eclipse. /12

During 1964 the author of this monograph proposed an observation method which to some degree replaces observations of the full moon [55]. The method involved photographing the moon in such a way that during the time of exposure (about one minute) there is an occultation of a star by the dark limb; in this case the proper motion of the moon must be compensated by motion of the photographic plate (Figure 1). Thus, a photograph is obtained of the seemingly motionless moon with stars moving about it. The line on the plate formed by the star occulted by the moon will be shorter than the line left on the plate by other stars not covered by the moon. The inner end of this trajectory coincides with the point of the true lunar contour at the point of occultation of the star by the moon. We can connect this point either with the crater

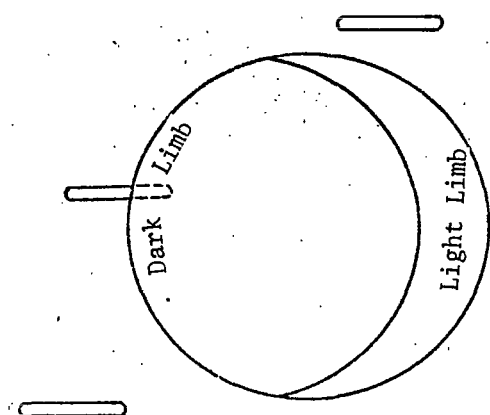


Figure 1

Moesting A and other craters, or with points on the visible lunar limb. In this case, at the same time it is necessary to register the times of onset and end of exposure with a high degree of accuracy. In processing the measurements it is necessary to take into account the radius of the star photographic image.

It can therefore be seen that over a long period of time we will obtain a considerable number of occultation points at

the dark limb, with simultaneous observation of the illuminated limb, i.e., we will have what in effect is photographs of the full moon in the entire range of libration variation.

If it is assumed that the smoothed visible lunar limb is represented well by a circle, it can be confirmed that: a) the radius of such a circle varies appreciably with optical libration, and b) the relationship between the center of a circle and the center of mass is not constant and does not even fall on the polar diameter. This is confirmed by the following. A. A. Yakovkin, for the first time in processing his own observations with the heliometer at the Engel'gardt Observatory, discovered that with an increase in optical libration in latitude the radius (reduced to mean distance) increases monotonically [43]. To be sure, reference is to the radius of a circle which does not coincide with the total lunar disk, but with the eastern or western half of the disk; the centers of these circles do not coincide. Corrections for relief in this study were taken from the maps compiled by Hayn. On the basis of heliometric observations (1916-1931), A. A. Yakovkin obtained $R = R_0 + 0''.048 \beta^0 \pm 0.006$ for a libration from -6.0 to $+7.6$. This shows that for each degree of optical libration the apparent lunar radius increases by $0''.05$. The value $0''.048$ can be called the "libration effect" coefficient in the lunar radius.

The libration effect has been studied by many researchers. I. V. Bel'kovich, in processing 150 heliometric observations, obtained a value $+ 0''.060$ [2]; A. A. Nefed'yev obtained $+ 0''.041$ [24] and $0''.057$ [26]. After comparing the lunar radii obtained in a Schluter series with the optical libration in latitude, A. A. Yakovkin found a value $+ 0''.090$ (without corrections for relief) as the libration effect coefficient [43]. T. Banakhevich, computing a Hartwig series (1890-1915), obtained for this same coefficient the following values: without allowance for relief corrections $+ 0''.054$; with allowance for relief corrections $+ 0''.029$ [60].

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The processing of Hartwig's Dorpat series, according to Koziel's computations, gives $+ 0''.040$ [79]. The cited data indicate that without exception all the series of heliometric observations reveal the presence of a libration effect in the lunar radius, provided the effect is determined separately from the western or eastern limb (without tie-in of the limbs).

Another shortcoming of Hayn's maps is the failure to take into account the influence of photographic halation. In order to investigate this effect Hayn photographed the lunar limb zones; as a result, he concluded that the effect of halation falls within the limits of accuracy in measuring altitudes.

According to investigations made by S. T. Khabibullin, in processing lunar photographs, the values of the photographic effect were insignificant [37]. He feels that the reason for this is the high contrast of the films employed. In [37] Khabibullin, in investigating the presence of a libration effect in the lunar radius, obtained for it a value $+ 0''.070 \pm 0''.018$.

In 1956, at the Main Astronomical Observatory Academy of Sciences Ukrainian SSR, using the 400-mm astrograph with an automatically moving cassette, I. V. Gavrilov [5] investigated the dependence of lunar radius on negative density. He obtained nine lunar photographs with identical exposures. The duration of development was varied in order to obtain negatives with different densities.

He obtained the following linear dependence of lunar radius on negative density:

$$\Delta h = -2''.33 + 0''.359 D \\ \pm 0''.18 \pm 0''.054$$

Similar investigations have been made by A. A. Gorynya and V. K. Drofa [13] in processing lunar photographs obtained at the Astronomical Observatory of Kiev State University by A. K. Osipov. Eight photographs were taken at the Astronomical Observatory of Kiev State University with an astrograph. These lunar photographs were taken with different sectoral diaphragms and without diaphragms. On each plate there were two lunar images photographed at short time intervals with different diaphragms. In addition, the camera was used in taking 26 photographs of an artificial "moon" with nonuniform illumination. Processing of these photographs revealed that there is a correlation between the negative density and lunar optical libration in latitude. Accordingly, it can be asserted that the use of lunar photographic images for studying the figure and rotation of the moon requires a more detailed study of the influence of photographic effects on the final observation results. The principal shortcoming in processing heliometric and other lunar observations is the lack of fully valid maps of the lunar limb zone. The maps compiled by Hayn contain systematic errors. The accuracy in determining the altitude of limb points from the Hayn maps is $\pm 0''.24$.

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For a long time these maps were the only ones which merited trust; not until 1954 did the French Astronomer Weimer [96] compile new maps, an atlas of lunar profiles. He used about 2,000 plates obtained by Levi and Puisier in 1894-1909 with the large equatorial instrument at the Paris Observatory. The diameter of the lunar image on these photographs is 17 cm. Since there are no stars or reference points on the photographs, orientation was performed relative to craters.

We will not discuss these maps, since they have been described in detail in a monograph by A. A. Nefed'yev [27]. We note only that the Weimer profiles provide nothing essentially new in comparison with the maps compiled by Hayn; they have the same shortcomings. The most important of these shortcomings is

that over the decades, from the time of Bessel to the present time, the profile of the lunar disk has been considered a circle. The accuracy in determining the altitude of relief at a particular point on the limb is estimated in this atlas to be characterized by a mean error from $0''.5$ to $0''.1$.

Hopmann pointed out [74] that the deviations from a circle do not exceed $1/5,000$ of the radius and have a random nature. All processing of observations was based on this conclusion. We will call this model Model "A" (spherical). It has been used as a basis in the first processing of all heliometric observations and in constructing relief maps. It was evidently assumed that deviations of the lunar disk figure from a circle are random and do not distort the lunar figure.

Lunar researchers have not even postulated that the centers of these most probable circles for each evening may not coincide with one another and that the linear radii of these circles are different, i.e., that the computed most probable circles do not belong to the same sphere and that a surface enveloping them in space cannot be constructed. No attempt has been made to clarify these problems, since it was assumed that small deviations from a circle can be neglected. So matters stood until 1932.

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However, even the first comparison of lunar radii found for different librations in latitude [43] revealed that there is an obvious correlation between the radius and optical libration. With a change in the lunar optical libration in latitude (from -7° to $+7^\circ$), the radius of the most probable circle, determined by the generally employed method, increases monotonically approximately by $0''.05$ for each degree of libration in latitude. With the simultaneous continuous displacement of the center of the circle in a southward direction, this phenomenon naturally caused a change in the lunar profile due to changes in optical libration.

The motion of the center of the most probable circle occurs relative to the center of mass. Since the position of the crater Moesting A is measured relative to the center of the most probable circle, which serves as the origin of a rectangular coordinate system, it is clear that the position of Moesting A relative to the center of mass in actuality does not change the same as

relative to the observed center of the lunar disk, which we will call simply the lunar center. Since the determined coordinates of the crater Moesting A served as initial data for all subsequent conclusions, it is clear that all conclusions will be erroneous if the described phenomena are not taken into account in the processing of observations.

It is becoming obvious that the use of a spherical model in processing lunar observations can be considered already outmoded and that it is necessary to find new approaches, new methods for solving this problem.

Search For New Approaches

Since the availability of good lunar limb zone relief maps is necessary for solving many problems involved in processing of lunar observations, this problem was and still remains urgent. The quality of the maps can be good only if their compilation is based on a correct concept of the lunar figure, i.e., if relief is related to a surface equidistant from the center of lunar mass, to the surface of a baricentric sphere. /17

It can now be considered demonstrated that all series of heliometric observations confirm the presence of a libration effect in the lunar radius (to be sure, if it is obtained from the eastern and western limbs separately).

As already mentioned, the beginning of a detailed study of the libration effect in the lunar radius was laid by A. A. Yakovkin in 1934 [43]. Later the libration effect was investigated by I. V. Bel'kovich [2]. After comparing the values of the radius separately for the western and eastern limbs, he arrived at interesting conclusions: 1) the apparent lunar radius is not dependent on optical libration in longitude for the two limbs; 2) the libration effect is considerably stronger for the western limb than for the eastern limb, and only in the series of observations made by A. A. Yakovkin is it almost identical. This can be attributed to the fact that in this series a third of the observations have an optical libration in latitude greater than $+6^\circ$; 3) the radius of the eastern limb in all series is greater than for the western limb, on the average by $0''.14 \pm 0''.03$ for mean optical libration.

I. V. Bel'kovich feels that it is established that there is a dependence between the lunar radius and optical libration in latitude and that the center

of the lunar disk figure for different optical librations does not maintain its position relative to the lunar center of gravity. He discovered a fluctuation in the crater longitude with a period $\omega/2\pi$, where ω is the perigee distance of the lunar orbit from the ascending node. The period of change in this parameter is 6 years. Based on a combination of the series obtained by Hartwig, Banakhevich, A. A. Yakovkin and I. V. Bel'kovich, the amplitude of the fluctuation is found to be $16'' \pm 5''$, whereas from the series obtained by A. A. Nefed'yev, it is found to be $32'' \pm 12''$. This amplitude is satisfactorily explained by the limb effect. In actuality, the apparent center of the lunar disks describes an ellipse on the lunar surface which has slowly rotating axes and a variable eccentricity with a period $\omega/2\pi$. Designating optical libration in latitude and longitude by λ_0 , β_0 respectively, the ellipse equation will be written, using the Banachiewicz formula as a point of departure [60], in the form

$$\frac{\lambda_0^2}{7.5^2} - 2\cos\omega \frac{\lambda_0\beta_0}{25} + \frac{\beta_0^2}{6.7^2} = \sin^2\omega.$$

It therefore follows that the lunar disk profile changes with a period of six years. If it is taken into account that optical libration in latitude is approximately equal to the ecliptic latitude multiplied by -1.4, it can be foreseen that the lunar radius determined from occultations of the Pleiades, having a latitude $+4^\circ$, will be less than determined from observations of occultations at the time of total lunar eclipses.

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Using eight occultations of the Pleiades, Peters obtained for the lunar radius a value $15'32''49$ (with $\beta_0 = -5.6$); Struve obtained $15'32''80 \pm 0.07$ (with $\beta_0 = 0^\circ$). It therefore can be seen that with a 1° change in libration the lunar radius is increased by $0''.05$. The reason for change in lunar radius can be attributed to the existence of an extensive plateau on the lunar surface in the high southern latitudes which extends around the polar region.

The mean altitude of this plateau increases if one moves along the central meridian from the invisible side of the moon through the pole onto the visible side. This plateau is asymmetrical in a longitudinal direction; as a result, the change in radius of the western and eastern lunar limbs as a function of libration is different, except for the case of large positive

librations. The hypothesis of "large relief" formulated by A. A. Yakovkin, explains clearly for the southern hemisphere the fact that the center of the lunar limb figure always is situated to the south of the center of mass by an average of 0".4. On this basis, A. A. Yakovkin adopted the following hypothesis (Model "B"): 1) the lunar contour is almost symmetrical relative to the central meridian, but does not have the form of a circle, but rather an egg-shaped curve. In formulating Model "B", the author used a curve for which the northern part is a semicircle, whereas the southern part is a semi-ellipse, whose semiminor axis coincides with the diameter of the lunar equator ($2R_0$), whereas the semimajor axis coincides with the lunar axis ($R_0 + \alpha$):

$$r = R_0 + a \cos^2 p,$$

where $a = 0$ when $270^\circ \leq p \leq 90^\circ$, p is an angle reckoned from the lunar north pole. This also is the equation for an ellipse with small flattening; 2) the center of lunar mass coincides with the center of the northern hemisphere. Thus, the lunar contour is defined by a curve with two parameters R_0 and α , with the second being a linear function of β_0 , i.e., the difference in the polar and equatorial diameters of the moon. The maximum altitude of the "large relief" above a sphere of the radius R_0 (at the end of the ellipse semimajor axis) is determined by the formula

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$$a = 0".96 + 0".082 \beta_0.$$

A. A. Yakovkin made an independent check of the libration effect in radius by studying its influence on the inclination of the lunar orbit to the ecliptic. If a libration effect exists, the inclination determined by the usual method will not be the true value.

We will assume that a large number of points, situated uniformly along some small circle and deviating from it in both directions in accordance with the law of random errors, is stipulated on the sphere. Computing the position of the great circle by the least squares method, smoothing out these points, we obtain a circle parallel to the small circle. Accordingly, in processing

meridian observations of lunar declinations it is possible to obtain a correct inclination of the lunar orbit for only one limb, even if an erroneous, but constant (in mean distance), radius is employed. In actuality, the computed points on the moon will be displaced from the great circle by the magnitude of the radius error, i.e., they will lie on the small circle parallel to the true lunar trajectory. If the lunar radius for the southern hemisphere is a linear function of optical libration in latitude, the inclination determined from observations of the southern limb will differ from the true value because the coefficients in the conditional equations, with allowance for a correction in the tabulated inclination Δi , correlate with libration in latitude.

In order to confirm his conclusions, A. A. Yakovkin partially processed lunar meridian observations at the Greenwich and Washington Observatories [47]. The computations were made separately for the northern and southern lunar limbs on the assumption that there is no libration effect in radius. The processing was done using conditional equations in the form

$$\frac{\sin(l-n)}{\sin \pi} x + y = l,$$

where $x = \Delta i$ is the correction for inclination of the lunar orbit to the ecliptic; y is the constant correction to lunar ephemeris radius.

Table 1 gives the values for the difference $\Delta i_S - \Delta i_N$.

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TABLE 1

Years	Series	$\Delta i_S - \Delta i_N$
1884-1889	Greenwich	+ 0".61
1923-1930	"	+ 0".42
1931-1935	"	+ 0".77
1879-1884	Washington	+ 0.63

It must be emphasized that these computations by A. A. Yakovkin were made on the assumption that the lunar radius is constant. Assuming the existence of a libration effect in radius, using the formula

$$\Delta R = \Delta R_0 + \delta \beta_0.$$

it is possible to determine the coefficient B from the condition $\Delta i_S = \Delta i_N$ and find the inclination corrected for the influence of the libration effect in the lunar radius.

Later, following a suggestion of A. A. Yakovkin, I. M. Demenko processed the Greenwich and Washington series of meridian observations. The processing included all observations of these series. Demenko also processed occultations of stars by the moon and their reappearances observed at the Cape Town Observatory [14] (Table 2).

TABLE 2

Years	Series	$\Delta i_S - \Delta i_N$	$\Delta i_w - \Delta i_e$
1923-1952	Greenwich	+ 0".45	--
1923-1948	Washington	+ 0.37	--
	Combined	+ 0.39	--
1881-1922	Cape Town I	+ 0.98	+ 0".06
	II	+ 0.72	+ 0.24
	III $\beta_0 < 0$	+ 1.13	- 0.08
	$\beta_0 < 0$	- 0.34	- 0.19

As shown by the results of processing by A. A. Yakovkin and I. M. Demenko, /21 the inclination determined from observations of the southern limb is always greater than the inclination determined from the northern limb. In addition (see Table 2), the libration effect is also determined from the western limb.

The processing of star occultations by the moon carried out by I. M. Demenko gave the correction for lunar latitude for the southern, northern, eastern and western lunar limbs [15] (Table 3).

TABLE 3

Variants	$\Delta B_S - \Delta B_N$	$\Delta B_w - \Delta B_e$
I	+ 0".36	+ 0".56
II	+ 1.29	+ 0.03
III $\beta_0 < 0$	+ 0.43	+ 0.66
$\beta_0 < 0$	- 0.55	+ 0.34

Table 3 shows that the corrections for lunar latitude for the southern limb are greater than for the northern limb, and for the western limb are greater than for the eastern limb. This confirms the presence of an asymmetry of the lunar disk not only relative to the central meridian, but also relative to the lunar equator.

Appropriate investigations were made for clarifying the existence of asymmetry of the lunar figure in longitude at Kiev during processing of lunar photographs taken at the Astronomical Observatory of Kiev State University (1950-1955) [13]. Computations made separately for the lunar eastern and western limbs gave corrections to the ephemeris radius, using the equation

$$\Delta r_0 + k\theta^0 + (\Delta r_D) \cdot D = \Delta r,$$

where Δr is the difference in the observed and computed radii, Δr_0 is the correction to the ephemeris radius, k is the coefficient for the dependence of the radius on optical libration in latitude (β^0), Δr_D is the change in radius in dependence on the negative blackening density D . For the eastern limb

$$\Delta r_0 = + 0".47; \quad k = + 0".21; \quad (\Delta r_D) = + 0.003;$$

for the western limb

$$\Delta r_0 = + 0".26, \quad k = + 0".06, \quad (\Delta r_D) = + 0.0031.$$

The collected data confirmed the presence of an asymmetry of the lunar figure in longitude. A. A. Nefed'yev, in processing his heliometric

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observations of 1938-1945 [26], also discovered an asymmetry in longitude:

$$h_{east} - h_{west} = 0''.14.$$

This is also indicated in a study by Spenser Jones [92], devoted to processing of observations of lunar star occultations made at the Cape of Good Hope. In this study he concludes that in the southeastern quadrant of the lunar disk there is a consistent elevation and that the mean correction for lunar ephemeris latitude, reduced to the eastern limb, is $- 0''.61$, whereas for the western limb it is $- 0''.28$.

On the basis of an analysis of all modern investigations, A. A. Yakovkin proposed a new model "C" closer to the true profile of the lunar limb zone [53]. This model is a semicircle for the northern hemisphere and a semi-ellipse with a variable semimajor axis, turned in the direction of a decrease in position angles by some angle γ (for the southern hemisphere). This model was investigated for the first time by A. A. Yakovkin and the author in [54] and was used in determining the parameters of lunar physical libration from the observations made by Hartwig (1890-1922). We note that model "C" explains the origin of the libration effect in the lunar radius [8]. In order to express better the local nature of the elevation, the southern semi-ellipse was replaced by a curve described by the equation

$$r = R_0 + a \cos^4 \varphi,$$

where $\phi = p - \gamma$; in model "C", $r - R_0 = a \cos^2 \phi$. It should be noted that in processing observations (when $\gamma = 20^\circ$) the mean square errors in the unknowns I and f were reduced by 30%.

Later the model "C" was used by I. M. Demenko [15] in processing Greenwich and Washington meridian observations of the moon and lunar star occultations at the Cape Town Observatory. He obtained a maximum thickness of the additional layer above the baricentric sphere of the radius $a = 1.22 + 0''.06 \beta^0$, which agrees well with the values obtained earlier by A. A. Yakovkin in processing heliometric and other observations.

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In checking existing lunar maps and models, Kh. I. Potter and N. F. Bystrov investigated the lunar figure by using lunar photographs obtained

during the full moon of 27-28 September 1958. The photographs were taken with the 26-inch refractor at the Pulkovo Observatory [32]. In checking the correctness of the model formulated by A. A. Yakovkin and the maps compiled by A. A. Nefed'yev and F. Hayn, as well as in determining the most probable lunar figure, they measured 360 radius vectors of limb points each 1° on a position circle. Employing the moving semicircles method, they concluded that the most probable figure of the lunar disk for a libration $\lambda_0 = + 2.^\circ 5$, $\beta_0 = - 1''.4$ must be an ellipse with a semimajor axis oriented at an angle of 36° to the lunar axis of rotation and with a flattening of 1:1,200; this corresponds to a difference between the semimajor and semiminor axes of $1''.5$.

On the basis of five lunar photographs taken at the Main Astronomical Observatory Academy of Sciences Ukrainian SSR with an astrograph ($D = 400$ mm, $F = 5.5$ m) with an automatically moving cassette, I. V. Gavrilov and A. N. Kur'yanova [6] made studies of the lunar disk figure. On these photographs they measured the radius vectors and position angles of points on the lunar limb (each 3°) relative to a mark (prick). In addition to the mentioned limb points, on two plates they measured the rectangular coordinates of eight craters whose selenographic coordinates had been taken from the Schrutka-Rechtenstamm catalogue [91]. In the five series of nonuniformities of the lunar limb, four were for the case of a libration close to zero, whereas one was for a case of extreme libration.

The resulting profiles were compared with profiles in the Weimer atlas [96] for corresponding librations. This comparison revealed a good agreement. In addition, the moving semicircles method was employed. When using this method and determining the parameters of the most probable ellipse, they discovered a small ellipticity of the lunar disk. I. V. Gavrilov and A. N. Kur'yanova feel that the representation of the lunar disk figure by the most probable ellipse is formalistic and inadequate and the irregularities should instead be reckoned from a circle whose center coincides with the projection of the lunar center of mass. However, determination of the lunar center of mass still remains an unsolved problem.

Processing of lunar photographs taken near a total lunar eclipse and described in [32, 6] can not give any information concerning the total lunar

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figure. Near a total lunar eclipse libration in latitude is close to zero. This material can be used in a special case for characterizing the lunar figure only in the case of libration in latitude close to zero. True, it can also be of interest in studying the libration effect in longitude, and also in investigating the lunar figure, since it is obtained (in the case of close librations in latitude) with a high amplitude of libration in longitude.

T. Weimer published an article entitled "Determining the Position of the Lunar Center and Elevation Reference Surface." Using all the profiles in the atlas compiled by T. Weimer, we obtained the mean profile for a spherical zone extending from $+7^\circ$ to -7° from a contour corresponding to the libration $L = 0^\circ$ and $B = 0^\circ$. On the averaged profile there are deformations in a zone with a width of 14° ; the author feels that these are more realistic than polar elongation (as given in the A. A. Yakovkin model). The best circle can be drawn and the deviation of curves which are 10 to 15 times less than in the "Yakovkin profile" from this best circle can be determined. T. Weimer asserts that if the "Yakovkin effect" exists, it should be manifested:

1) in lunar positions determined from occultations or from meridian observations. It should be possible to detect an appreciable discrepancy between the observations made on the eastern and western limbs. This is actually the case. Markowitz has pointed out that the lunar longitude correction determined from observations of its eastern limb is $1''.2$ greater than the correction determined from observations on the western limb. I. V. Bel'kovich, on the basis of five large series of observations, determined the lunar radius separately for the eastern and western limbs. In all series the mean radius of the eastern limb was greater than for the western limb. The mean difference was $0''.14 \pm 0''.03$. The correction in lunar latitude also differs greatly for the eastern and western limbs [84].

2) in computations of physical libration, based on observations, if the eastern and western lunar limbs are examined separately. We feel that this will be true, although such investigations still have not been described in the literature; /25

3) in the relative position of the centers and in the length of the radii for a semiprofile, i.e., for a quarter of a circle. However, it has now been demonstrated that the position of the figure (circle) center relative to the center of mass is dependent on optical libration, i.e., it continuously changes. Thus, none of the three objections cited by Weimer contradict the A. A. Yakovkin model "A", but rather are a confirmation of his work.

In addition, a study by I. M. Demenko [15] gives investigations of the dependence of the lunar polar diameter on libration in latitude and the horizontal diameter on libration in longitude, determined from Greenwich meridian observations. The processing results reveal that the vertical diameter is dependent on topocentric libration in latitude ($+ 0''.121$) and the lunar equator differs from a circle; there is also an east-west asymmetry relative to the central meridian. The changes in the horizontal diameter correlate with libration in longitude and the maximum changes by $0''.705$. The lunar maps and limb zone profiles compiled by Hayn, Przybyllok, Weimer, and A. A. Nefed'yev constitute a considerable contribution to study of the lunar figure.

The maps and profiles constructed by the first three of the mentioned authors were based on the assumption that the centers of the most probable circles coincide. The presence of asymmetry in the lunar figure, discovered by A. A. Yakovkin, gives rise to a number of questions concerning the structure of the mentioned maps. What form does the zero surface for these maps have? What is the equation used for this surface by each of the mentioned authors? How and by what is the zero surface related to the center of mass, etc.? However, on the other hand, the existence of asymmetry can be taken into account when using these maps if tables are compiled which, using the arguments P and D , give a reduction to the baricentric zero surface.

A. A. Nefed'yev constructed lunar limb zone maps with allowance for the moon's asymmetry relative to the lunar equator; this was a considerable advance. Although these maps do not take into account lunar asymmetry relative to the axis of rotation, essentially they were constructed for a common zero level. In 1961 A. A. Nefed'yev [28] made a study of these profiles and maps, using for this purpose the series obtained by Krasnov, Hartwig's Bamber series [67], and his own series. In order to check the assumption that the radius of the

most probable circle, corrected for irregularity, determined from these maps, should not be dependent on optical libration in latitude, the observations were corrected by an allowance for lunar limb corrections read from these maps and profiles. The measured radius vectors

$$\Delta x_0 \cos \rho + \Delta y_0 \sin \rho - \Delta R = R_0 - R$$

were corrected for limb irregularity. The least squares method was used in determining corrections to the ephemeris radius ΔR . After comparing the determined ΔR with optical libration in latitude (B), A. A. Nefed'yev wrote the equations

$$\Delta R = \Delta R_0 + k\delta,$$

from which he determined ΔR_0 (correction to radius) and the coefficient of the libration effect in radius (k).

TABLE 4

Series	Cor- rec- tion	Maps Compiled by F. Hayn	Profiles Con- structed by E. Przybyllok	Profiles Con- structed by T. Weimer	Maps Compil- ed by A. A. Nefed'yev
Krasnov .	ΔR_0	$-1''.38 \pm 0''.05$	$-1''.39 \pm 0''.07$		$-1''.38 \pm 0''.04$
	k	$+0.057 \pm 0.013$	$+0.028 \pm 0.015$		-0.008 ± 0.010
Hartwig	ΔR_0	$+0.45 \pm 0.03$			$+0.57 \pm 0.03$
	k	$+0.034 \pm 0.008$			$+0.001 \pm 0.001$
Nefed'yev	ΔR_0	-0.51 ± 0.05		$-0''.52 \pm 0''.07$	-0.44 ± 0.05
	k	$+0.041 \pm 0.013$		$+0.048 \pm 0.016$	-0.004 ± 0.011

The data given in Table 4 convincingly demonstrates that on the Hayn maps and the Przybyllok and Weimer profiles no allowance was made for the libration attacks. With respect to the maps compiled by A. A. Nefed'yev, the mentioned effect is not reflected on them. This indicates a need for taking into account the libration effect in lunar radius when compiling maps of limb zone relief. In 1946 the United States Naval Observatory compiled new maps. About 700 lunar photographs obtained on 503 nights during the years 1927-1956

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were used for this purpose. These photographs were taken at the United States Naval Observatory, the Colombia Station of Yale University, at Johannesburg, and at the Lowell Observatory. The photographs were measured with a photo-electric instrument with graphic recording in the form of profiles at a scale of 41 mm per degree on a position circle and 16 mm per second of arc in radius. In order to obtain the configuration of the spherical surface, each profile was represented by an arc of a circle. In order to improve the zero (reference) surface, Watts selected 50 lunar photographs as references. These photographs were used in constructing profiles at the points of intersection of limb lines; he found the corresponding corrections, and used them in constructing the zero surface.

Photographs taken near the full moon were used in determining a preliminary contour which was close to an ellipse with a semimajor axis oriented in a position angle of 153° . This axis (polar) exceeds the semiminor axis by $1''.2$. In order to exclude the observed ellipticity, Watts shifted the determined center of the zero surface by $0''.30$, thereby correcting all 46,700 measured radii. In 1963 he published 1,800 individual map-graphs [95]; each of these showed the elevations for a definite position angle. The contours were drawn at $0''.2$ intervals. The mean error in limb correction was $+ 0''.1$, i.e., 200 m on the lunar surface.

Watts attempted to represent the level surface in the form of a sphere. In checking this, A. A. Nefed'yev [28] used his own heliometric observations made during 1938-1945. After correcting his observations from the maps compiled by Watts, he wrote conditional equations (also for checking the maps compiled by Hayn, and others). Solution by the least squares method gives a correction ΔR to the ephemeris radius and R_0 and the libration effect coefficient. The determined ΔR values were grouped by optical libration in latitude values and the following sought-for values were obtained:

$$\begin{aligned}\Delta R_0 &= - 0''.689 \pm 0''.032, \\ k &= + 0''.037 \pm 0''.008.\end{aligned}$$

The presence of a libration coefficient in the lunar radius indicates that the Watts maps give elevations of points on the lunar surface from some still unknown level which differs from a sphere.

In 1967 D. P. Duma and A. S. Duma (Main Astronomical Observatory Academy of Sciences Ukrainian SSR) published an article entitled "Some Results of a Study of the Watts Maps" [20]. This was done by processing Washington meridian observations of the moon for the period 1956-1962, with introduction of corrections for lunar limb irregularity as indicated on the Watts maps. As a result, they obtained:

1) corrections to the orbital elements

$$\begin{aligned}\Delta\lambda &= + 17''.97 \pm 0''.02, & \sin i \Delta R &= + 0''.08 \pm 0''.05, \\ \Delta e &= - 0''.00 \pm 0''.01, & \Delta\delta_0 &= - 0''.28 \pm 0''.05, \\ 2e\Delta\pi &= - 0''.24 \pm 0''.02, & \Delta R &= + 0''.05 \pm 0''.05, \\ \Delta i &= - 0''.35 \pm 0''.05,\end{aligned}$$

Displacement of the observed center relative to the ephemeris center

$$\Delta\beta = - 0''.03 \pm 0''.04;$$

2) the libration effect in the lunar radius

$$R = R_0 = 0''.058\beta \pm 0''.002;$$

3) the dependence of the correction in lunar longitude $\Delta\lambda$ on the position angle of the lunar axis (C)

$$\Delta\lambda = 17''.19 \pm 0''.021C^\circ$$

The authors conclude that the Watts maps have some advantage over the Hayn maps in that they eliminate the latitude correction ($- 0''.5$ -- $- 0''.7$), and also there is no "jump" in lunar longitude with transition from the eastern to the western limb. However, a libration effect in radius exists.

That same year L. N. Miz' (Main Astronomical Observatory Academy of Sciences Ukrainian SSR) also analyzed the Watts maps using the heliometric observations made by Schluter (1841-1843) [23]. The Schluter series had been processed earlier by Wichmann and Franz without any maps. Simultaneously with

computation of optical librations in longitude and in latitude (l and B), he determined the Hayn limb coordinates P and D ; this later made it possible to make a comparison with the Hayn maps.

The determined corrections for limb irregularities were introduced into the measured distances s . The deviations in radius Δh_0 were combined into groups on the basis of optical libration in latitude β_0 . In solving the system of conditional equations $\Delta h_0 = \Delta h + k\beta_0$ for the libration coefficient in lunar radius, a value of $+0''.05$ was obtained. This shows that the correction of lunar observations on the basis of the Watts maps does not exclude the libration effect in radius, i.e., the new maps were not constructed on a single spherical surface. The profile irregularities of the apparent lunar limb are reflected strongly in position observations. In order to take them into account, Hayn, Przybyllok, Weimer, A. A. Nefed'yev and Watts used data from observations for which either the eastern or western half of the lunar disk was visible; they compiled maps of the lunar limb profile. Accordingly, it is of considerable interest to examine lunar photographs taken during annular solar eclipses. Such photographs make possible a simultaneous study of the entire lunar disk limb.

An interesting result was obtained by American expeditions in Africa during the annular solar eclipse of 31 July 1962 and that of 25 January 1963. The lunar limb profile was constructed from these photographs. It was found that the lunar limb can be represented by an ellipse with a flattening of about 1/1,000; the elliptic component has a semimajor axis which is oriented at an angle of 37° to the lunar polar axis. These results agree well with the results obtained by Kh. I. Potter.

The Main Astronomical Observatory of the Academy of Sciences Ukrainian SSR organized an expedition for observing the annular solar eclipse of 20 May 1966 [16]. The observations were made with 10- and 12-meter coronagraphs. The resulting photographs of the annular phase were used for determining the geometric figure of the lunar profile. The radius vectors of the rings and crescents were reduced to a uniform system for both sectors of the lunar limb having rings and crescents on the photographs. The reduced width of the ring and the radius vector were expanded in a Fourier series.

The elliptic components for the total profile agree approximately with the results of studies made by the American expedition, Kh. I. Potter, and the profile on the Watts maps. The coefficients in the expansions of the Fourier series cannot be explained by an ellipse model; accordingly, the authors of [16] proposed a different curve which gives a better representation of the lunar profile: Pascal's limaçon.

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CHAPTER II

REVIEW OF STUDIES ON DETERMINATION OF LUNAR PHYSICAL LIBRATION PARAMETERS

It is now well-known that the lunar configuration is nonspherical. It is obvious that the moon was once liquid and that in the hardening process it assumed a stable configuration. An influence was asserted by the earth; our planet could cause a tide on both the visible and far sides of the moon. This tide was 180 times greater than the tide in terrestrial seas caused by the moon.

It has now been established that the moon is elongated in the direction of the earth; possibly it is symmetrical in the opposite direction. The earth-moon system is regarded as a double planet whose center of gravity is situated at a distance of 4,672 km from the center of the earth. The earth determines the Keplerian orbit of the moon (i.e., the value of the mean diurnal motion in longitude, etc.).

It is the sun which is the principal perturbing celestial body in lunar orbital motion. The greatest perturbations in lunar longitude are evection ($1^{\circ}16'$) and variation ($39'$), caused by the sun. In addition, there are small perturbations in longitude which are caused by flattening of the terrestrial ellipsoid. The principal law of lunar rotation is that the mean time of its rotation about its axis is equal to the period of its revolution in orbit about the earth.

Galileo, studying the lunar surface through a telescope, was the first to discover that features on the lunar surface periodically move relative to the limbs of its apparent disk. (This phenomenon was later given the name optical libration.) In 1610 he discovered two types of libration: diurnal and libration in latitude. The first is a result of lunar parallax and varies with zenith distance; the second is dependent on the position of the moon relative to the nodes of its orbit in the plane of the ecliptic. However, this phenomenon was not explained. In 1648 Hevel and Riccioli discovered a third

type of libration, libration in longitude. Later the second and third types of libration were called optical libration.

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Newton (1687) explained this phenomenon of optical libration as the result of nonuniform motion in an elliptical orbit during uniform rotation about its axis. More precise laws of lunar rotation, describing optical libration, were formulated in 1693 by a contemporary of Newton, Dominick Cassini.

1. The moon rotates with a constant angular velocity about an axis which maintains its position constant in the lunar body.

2. The inclination of the lunar equatorial plane to the plane of the ecliptic is a constant value.

3. The poles of the lunar equator, ecliptic and lunar orbit lie on a single great circle; accordingly, the longitude of the ascending node of the lunar equator on the ecliptic is equal to the longitude of the descending node of the lunar orbit in the ecliptic.

These laws very precisely reflect the true lunar rotation. Observations have confirmed only insignificant deviations which had been discovered theoretically by Lagrange.

Almost 50 years later (1748-1749), on the basis of his own observations of four lunar craters, Tobias Mayer determined the inclination of the lunar equator to the plane of the ecliptic (I), equal to $1^{\circ}32'$ (Cassini obtained $2^{\circ}30'$ for this same parameter). He was the first to introduce a system of selenographic coordinates and give a determination of the principal (zero) meridian. This system is still used.

Forced Lunar Physical Libration

Many studies of outstanding mathematicians have been devoted to the lunar rotation and motion. In 1747 Clairaut pointed out ways in which this problem could be solved by applying the methods of mathematical analysis.

The studies of Euler (1753) validated the application of the method of variation of arbitrary constants (the osculating orbit method) for formulating a theory of the moon; this was later developed in studies by Delaunay

(1846); he formulated a theory of solar inequalities in lunar motion. In addition, he advanced the idea of expanding lunar coordinates in series, arranged in powers of the eccentricities of the lunar and solar orbits and inclination of the lunar orbit to the ecliptic. Later this idea was used by Hill, and then by Brown, for compiling lunar tables and ephemerides. The unsuccessful attempts of d'Alembert to give a theoretical explanation of Cassini's laws also dates to this time.

In 1763 Lalande confirmed Cassini's laws by observational data. He found that $I = 1^{\circ}43'$. In 1764 Lagrange, in formulating a theory of lunar physical libration, suggested that Cassini's laws do not describe true lunar rotation. It is well known that lunar rotation can be described by the change in three Euler angles, ϕ , θ and ψ ; then Cassini's laws are expressed by the formula $\phi = 180^{\circ} + m - n$; $\theta = I$, $\psi = n$.

Lagrange formulated the objective of deriving Cassini's laws from gravitational laws. In 1780 he discovered that these laws are approximate and that there is a number of small variable terms which must be added to the Euler angles, i.e., $\phi = 180^{\circ} + m - n + \tau$, $\theta = I + \rho$, $\psi = n + \sigma$. These additional terms were called physical libration in longitude (τ), in inclination (ρ) and in node (σ). The presence of these terms, according to Cassini's laws, was caused by the perturbing effect of the earth and sun on uniform lunar rotation. Thus, Lagrange was the first to discover the difference in actual lunar rotation from Cassini's laws. The theory of lunar physical libration was developed further in studies by Laplace [80, 82]; he developed the theory to the point of solving the fundamental equations which are still used at the present time without significant changes.

Lunar physical libration consists of forced libration (dependent on the influence of the earth and sun) and free libration (dependent on the "initial conditions" of rotation): noncoincidence of the lunar axis of rotation with the axis of inertia. The division of oscillations into "forced" and "free" follows because the problem was reduced to linear form. The value of free libration and even its existence is questioned by some researchers even at the present time.

Thus, theory not only explained Cassini's laws, but even made them more precise: physical libration was discovered. This type of libration, naturally, /33 had to be confirmed by observations.

In accordance with a proposal made by Laplace, at the Paris Observatory Bouvard, Arago and later Nicollet made special lunar observations (1806-1810) using an instrument with $F = 1$ m, $D = 65$ cm. Later, from 1819 through 1820, the observations were made only by Nicollet. On the basis of his collected data, Nicollet determined the inclination of the lunar equator to the ecliptic $I = 1^{\circ}28'45''$ and the value of the function of the lunar moments of inertia $f = 0.055$.

In addition to Bouvard, Arago and Nicollet, during the period 1831 to 1834 observations were made at Milan by Crayle and Stambucchi, who obtained $I = 1^{\circ}33'48''$ and $f = 0.005$. However, all the recomputed observations failed to give reliable values due to the low accuracy.

Then the problem arose of finding new methods for increasing observational accuracy. Such a method, which could be regarded as ideal for that time, was found by the famed astronomer Bessel [3]. He used a new instrument, a heliometer, for lunar observations. In making observations with a heliometer distances and position angles of several selected points on the lunar limb are measured relative to some crater, selected as a permanent reference, situated near the center of the lunar disk. Bessel proposed observations of the crater Moesting A; this is situated near the center of the lunar disk ($- 5^{\circ}11'$ E and $- 2^{\circ}12''$ S) with an apparent diameter of about $5''$.

Some observers proposed that the crater Moesting A be replaced by some other crater closer to the lunar center, such as Triesnecker B. Nevertheless, further observations were made specifically with the crater Moesting A. On the basis of his own experience, the author feels that the crater Moesting A is more suitable than any other situated near the center of the visible lunar disk because it stands out clearly among the other craters in its neighborhood. It stands out brightly and in detail against the lunar surface and it can be observed with a greater accuracy than the others.

The method proposed by Bessel for measuring distances of a marked point on the limb from the crater Moesting A involves bisecting of the crater image /34

by the lunar limb. Such measurements have been made at 7 to 11 points situated approximately uniformly along the illuminated lunar limb. In proposing this observation method, Bessel derived detailed formulas for determining the parameters of lunar physical libration. He was the first to make heliometric observations for obtaining the selenographic coordinates of the crater Moesting A. The observations were made with the first fabricated heliometer (old design) at the Koenigsberg Observatory.

In 1841-1843, working at Koenigsberg, Schluter (a student of Bessel) made the first heliometric observations, involving 158 measurements of the crater Moesting A; it must be said that these measurements were made with an accuracy exemplary for that time. This series of observations is of interest to researchers even at the present time.

After Schluter, observations were continued by Wichmann. During the years 1840-1846 he made 50 measurements of the crater Moesting A. In 1848 he completed processing of his own observations [97]. These were the first reliable results of a study of the parameters of lunar physical libration:

$$\begin{aligned}\lambda &= -5^{\circ}13'23'' \pm 89'', & I &= 1^{\circ}32'09'' \pm 249'', \\ \beta &= -3^{\circ}10'55'' \pm 24'', & f &= 0.48 \pm 0.18.\end{aligned}$$

These observations were processed without allowance for irregularities of the lunar limb; the author explains that this was done due to the large errors in determining the unknowns. In this study he made a large contribution to the development of concepts concerning lunar figure and motion. He improved the Laplace theory; this made possible its direct application for the processing of observations. He brought attention to the fact that the coefficients for inclination and for the amplitude of free libration in the conditional equations which are contained in the equations for determining the elements of lunar rotation are almost proportional. This is one of the reasons for their poor discrimination.

Further heliometric observations were made with the Strassburg heliometer by Hartwig. During the years 1877-1879 he made 42 observations of the crater Moesting A. A year later he published the results of processing observational

data [67] by the Bessel-Wichmann method. After determining more precisely the elements of lunar physical libration, he determined the value of the parameters both from his own observations, and from the observations made by Wichmann. /35

Wichmann Series

$$\begin{aligned}\lambda &= 5^{\circ}13'22'' \pm 50'', \\ \beta &= -3^{\circ}11'03'' \pm 17'', \\ I &= 1^{\circ}32'39'' \pm 256'', \\ f &= 0.357 \pm 0.102.\end{aligned}$$

Hartwig Series

$$\begin{aligned}\lambda &= 5^{\circ}10'58'' \pm 24'', \\ \beta &= 3^{\circ}10'23'' \pm 12'', \\ I &= 1^{\circ}36'39'' \pm 139'', \\ f &= 0.507 \pm 0.060.\end{aligned}$$

We see that the accuracy in determining the unknowns in the Wichmann series was considerably increased.

During the years 1884-1885, Hartwig made a second series of observations, but in this case at Dorpat (Tartu) Observatory, also using an instrument of old design. This series contains 36 observations.

Late in 1885 Hartwig was appointed director of the Bamberg Observatory; he acquired a heliometer of a new design for this observatory. This new Repsold instrument had $D = 184$ mm, $F = 2.7$ m, whereas the "old" heliometers had $D = 106$ mm, $F = 1.7$ m. Using this instrument, working during the years from 1890 through 1923, he made observations in a third (and the last) series, consisting of 266 measurements of positions of the crater Moesting A.

Developing and improving the theory of lunar physical libration, Franz reworked the second Hartwig series with different initial I and f values ($I_0 = 1^{\circ}35'46''.8$, $f_0 = 0.489$) [65] and obtained:

$$\begin{aligned}\lambda &= -5^{\circ}11'10'' \pm 16'', & I &= 1^{\circ}31'36'' \pm 17'', \\ \beta &= -3^{\circ}10'07'' \pm 13'', & f &= 0.468 \pm 0.056.\end{aligned}$$

Thereafter, Franz published [66] the results of processing of the Schluter series, at the same time improving the theory of processing of observations. He obtained the following values:

$$\begin{aligned}\lambda &= -5^{\circ}10'19'' \pm 8'', & I &= 1^{\circ}31'22'' \pm 7'', \\ \beta &= -3^{\circ}11'24'' \pm 6'', & f &= 0.488 \pm 0.028.\end{aligned}$$

As we see, the accuracy in determinations in this case was high.

Subsequent fundamental studies devoted to this problem were made by Hayn. After setting about to determine the selenographic coordinates of craters, he concluded that the problem can be successfully solved provided that the parameters of lunar physical libration are well known. Accordingly, he first concerned himself with theoretical problems [68]. He investigated the influence of the sun on lunar rotation; this was found to be negligible. Hayn was the first to discover the critical value of the function of the lunar moments of inertia $f = 0.662$. In 1904 he published the results of observations of 55 base craters which he had made during the years 1898-1903 at Leipzig [69].

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Like Wichmann and Franz, Hayn determined the parameters of lunar physical libration. In the conditional equations he introduced a correction for lunar radius for five points; it was this which distinguished his method for processing observations from the method employed by Wichmann and Franz. He discovered that the crater Moesting A is situated 2-3" more distant from the lunar center. As a result, in our opinion, he obtained very interesting results for the parameter f .

Using as the initial values $I_0 = 1^\circ 31' 22''$ and $f_0 = 0.50$, he obtained $I = 1^\circ 32' 3$ and $f = 0.85$. Hayn notes that the results can be different if deviations of the lunar limb from a circle are taken into account.

In order to clarify the differences in the results obtained by Franz and Hayn, Stratton [93] reworked the Schluter series, with an allowance for a correction for lunar radius which had been discarded by J. Franz. He obtained values close to the results obtained by Franz, specifically: $f = 0.50 \pm 0.03$.

In 1907 Hayn published a third major study [70] giving the results of determinations of the parameters of physical libration from the second series of Hartwig's observations and his own observations. He was convinced of the need to take into account corrections for limb zone relief. Hayn determined the corrections for limb irregularities from observations made by Hartwig

and himself. After correcting the observations for limb irregularities, he obtained the following values:

Hartwig Series	Hayn Series
$\lambda = -5^{\circ}10'16'' \pm 23''$,	$\lambda = -5^{\circ}10'08'' \pm 19''$,
$\beta = -3^{\circ}10'31'' \pm 21''$,	$\beta = -3^{\circ}11'09'' \pm 14''$,
$h = 15'34''76 \pm 1''18$,	$h = 15'34''57 \pm 0''64$,
$I = 1^{\circ}31'53'' \pm 36''$,	$I = 1^{\circ}32'14'' \pm 22''$,
$f = 0.71 \pm 0.08$,	$f = 0.75 \pm 0.04$.

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Thus, the parameters of lunar physical libration were obtained for the first time with allowance for corrections for limb irregularities, although on the basis of inadequately extensive data. The determined values caused Hayn to proceed to compilation of detailed maps of the lunar limb zone. During the period 1908-1911 he took 50 lunar photographs with the refractor at Leipzig Observatory for this purpose. These photographs were used in compiling lunar profile maps.

Having constructed the limb zone maps, Hayn undertook a more precise determination of the parameters of lunar physical libration, he again computed the second Hartwig series for this purpose, together with his own Leipzig series, with the new maps taken into account, as well as the Schluter series (in the new processing performed in 1913 by Naumann) and obtained the following values:

Schluter Series	Hartwig Series	Hayn Series
$\lambda = -5^{\circ}09'56'' \pm 11''$,	$\lambda = -5^{\circ}10'26'' \pm 25''$,	$\lambda = -5^{\circ}10'10'' \pm 20''$,
$\beta = -3^{\circ}01'00'' \pm 9''$,	$\beta = -3^{\circ}10'32'' \pm 21''$,	$\beta = -3^{\circ}11'15'' \pm 14''$,
$h = 15'32''.9 \pm 0''.44$,	$h = 15'34''.7'' \pm 1''.15$,	$h = 15'33''.9 \pm 0''.66$,
$I = 1^{\circ}32'14'' \pm 14''$,	$I = 1^{\circ}31'51'' \pm 36''$,	$I = 1^{\circ}32'33'' \pm 15''$,
$f = 0.71 \pm 0.03$,	$f = 0.73 \pm 0.07$,	$f = 0.77 \pm 0.04$.

After combining these three series, Hayn obtained

$$\begin{aligned} \lambda &= -5^{\circ}10'07'' \pm 9'', & I &= 1^{\circ}32'20'' \pm 10'', \\ \beta &= -3^{\circ}11'02'' \pm 7'', & f &= 0.73 \pm 0.02. \\ h &= 15'33''.4 \pm 0''.34, \end{aligned}$$

The latter values were regarded as reliable and used for computing the ephemeris for the crater Moesting A in all national almanacs. The third Hartwig series of observations, the largest for this time, for a long time remained unprocessed.

In 1939 Naumann [85] processed 157 observations of this series, with and without taking into account limb irregularity; he obtained:

With allowance for relief	Without allowance for relief
$\lambda = -5^{\circ}09'16'' \pm 15''$,	$\lambda = -5^{\circ}10'00'' \pm 11''$,
$\beta = -3^{\circ}10'28'' \pm 11''$,	$\beta = -3^{\circ}10'56'' \pm 9''$,
$h = 15'36.9 \pm 0.97$,	$h = 15'34.2 \pm 0.41$,
$l = 1^{\circ}30'19'' \pm 17''$,	$l = 1^{\circ}31'28'' \pm 12''$,
$f = 0.72 \pm 0.03$,	$f = 0.71 \pm 0.03$.

We see that these values are close to the results obtained by Hayn.

In 1917 a study by Johnson [76] was published. It was devoted to the theory of lunar physical libration. It employed equations of lunar motion in canonical form, solved by the Charlier method. The author also demonstrated how it is possible to expound the fundamental problems in celestial mechanics relative to lunar rotation. The rotation of a solid spherical body is regarded as an "intermediate path". In this case the six determined integration constants can be used as variables and it is possible to take into account the deviations from a spherical configuration and external forces. Charlier demonstrated that in the problem of planetary rotation there are some special integrals which should be used as an initial solution in a study of lunar rotation.

In his study Johnson investigated lunar rotation, making the assumption that the moon is a solid body, that is, precisely by the use of this method. The author uses the derived formulas for computing inequalities in lunar rotation arising due to the earth's influence.

The conclusions drawn by Johnson gave the same result as those drawn by Hayn, but the sign for the term with the argument 2ω in the expansion of physical libration in longitude τ in Johnson's work was the opposite. In his studies, Hayn used tables compiled by Hansen, whereas Johnson used the tables

compiled by Brown. However, this obviously could not exert any effect on the final results. It must be emphasized that after publication of the article by Hayn [73], devoted to the results of his many years of work in the field of theory and in determining the parameters of lunar rotation, scientists in no other country, other than Russia, worked so productively in studies of lunar rotation. Kazan became a center for investigating the parameters of lunar physical libration. We will not discuss in detail the history of the heliometric observations at Kazan', since this has been done very thoroughly in studies by A. A. Nefed'yev [27] and Sh. T. Khabibullin [37]. We will only /39 note the principal directions in the investigations.

The initiative for acquiring a heliometer and organizing observations with such an instrument was taken by D. I. Dubyago, Director of the Kazan' Observatory (1884); however, it was not until 1895 that regular observations were initiated by A. V. Krasnov. During the years 1895-1898 he made 112 observations of the crater Moesting A and 50 observations of other craters.

After the departure of A. V. Krasnov from Kazan' Observatory, a second series of observations with this instrument was made by A. A. Mikhaylovskiy; during the period 1900-1905 he made 54 observations of the crater Moesting A. However, these two series were not processed. Only in 1908 did M. Fel'kel', a specialist at the Breslau Observatory, process a series of observations made by A. A. Mikhaylovskiy [98], without taking into account irregularities in the lunar limb zone.

The following were the first results of determination of the parameters of lunar physical libration on the basis of Kazan' heliometric observations:

$$\begin{aligned} \lambda &= -5^{\circ}12'32'' \pm 32'', & I &= 1^{\circ}31'46''.5 \pm 39'', \\ \rho &= -3^{\circ}10'14'' \pm 28'', & f &= 0.79 \pm 0.11. \end{aligned}$$

In 1908 the heliometer was somewhat modernized by the Repsold Company and set up outside the city, at the Engel'gardt Observatory.

During the period 1910-1915, T. A. Banachiewicz made a third series of observations, this time at the Engel'gardt Observatory, consisting of 130 measurements of the crater Moesting A. After his departure from Kazan',

observations were continued by A. A. Yakovkin; he devoted all his scientific activity for the most part to a study of lunar rotation and motion. He is the founder of the Kazan', and later the Kiev Schools for studying lunar rotation and figure.

Beginning in the 1920's, the Engel'gardt Astronomical Observatory became the leading institution in the field of lunar rotation studies. Major investigations were made here by A. A. Yakovkin, I. V. Bel'kovich, A. A. Nefed'yev, and Sh. T. Khabibullin.

In 1928 A. A. Yakovkin [42] published data based on processing of a Banachiewicz series. This was the first work in the Russian language devoted to the study of lunar rotation. Observations were processed with a detailed theoretical foundation for both the problem itself, and the observational reductions which were taken into account. The work became a standard reference not only for Kazan' astronomers, but also for astronomers at other Soviet Observatories who subsequently took up these problems involved in study of lunar rotation. The observations were corrected for relief irregularities of the lunar limb zone on the basis of Hayn's maps. Since a full lunar observation by the Bessel method frequently requires more than an hour and the crater Moesting A during this time moves a significant distance along the lunar disk, new, precise formulas were written and employed for reducing observations to a selected mean time. The formulas for computing P and D were newly derived in this study and represented in simpler and more convenient form for use. /40

Individual conditional equations were assigned weights and normal equations were written separately for longitude and latitude. Since the latitude equations are more precise than the longitude equations, after two preliminary solutions with determination of the sum of the squares of deviations, the latitude equations were multiplied by a factor 1.9608 and combined term-by-term with the longitude equations. The solution of the normal equations derived in this way gave the values:

$$\begin{aligned} \lambda &= -5^{\circ}10'19'' \pm 11'', & I &= 1^{\circ}32'02'' \pm 17'', \\ \beta &= -3^{\circ}10'56'' \pm 10'', & f &= 0.74 \pm 0.03, \\ h &= 15'32.75 \pm 0.75, \end{aligned}$$

These are essentially the first results obtained in the Soviet Union on the basis of extensive observational data. We see that the f value was derived with a good accuracy and is close to that obtained by Hayn.

In proceeding to observations with the heliometer at Engel'gardt Observatory in 1916, A. A. Yakovkin continued these observations until 1931; he made 251 measurements of the crater Moesting A. The first 200 observations of this series were processed by Yakovkin and published in 1939 [44]; this publication gives the values of the sought-for parameters:

$$\begin{aligned} \lambda &= -5^{\circ}12'29'' \pm 11'', & I &= 1^{\circ}31'36'' \pm 14'', \\ \beta &= -3^{\circ}10'19'' \pm 8'', & f &= 0.71 \pm 0.03, \\ h &= 15'34.73 \pm 0.45, \end{aligned} \quad /41$$

The remaining 51 observations were processed by A. A. Yakovkin in 1945 [45];

$$\begin{aligned} \lambda &= -5^{\circ}10'07'' \pm 32'', & I &= 1^{\circ}32'18'' \pm 37'', \\ \beta &= -3^{\circ}10'19'' \pm 27'', & f &= 0.59 \pm 0.09, \\ h &= 15'33.75 \pm 1.1, \end{aligned}$$

From a joint solution of normal equations for all (251) observations he obtained:

$$\begin{aligned} \lambda &= -5^{\circ}10'25'' \pm 10'', & I &= 1^{\circ}31'44'' \pm 12'', \\ \beta &= -3^{\circ}10'26'' \pm 8'', & f &= 0.68 \pm 0.02, \\ h &= 15'34.71 \pm 0.4, \end{aligned}$$

After A. A. Yakovkin had completed his observations, work with the heliometer was continued by his student I. V. Bel'kovich; during the years 1932-1948 he made 247 observations of the crater Moesting A. The studies of I. V. Bel'kovich are an important contribution to study of lunar rotation. In 1936 he published a processing of a series made by A. A. Mikhaylovskiy [1]; he gave interesting results which differ from all preceding investigations (other than the data obtained by Hayn in processing his first series of observations),

$$\begin{aligned} \lambda &= -5^{\circ}12'42'' \pm 28'', & I &= 1^{\circ}33'10'' \pm 32'', \\ \beta &= -3^{\circ}11'24'' \pm 19'', & f &= 0.84 \pm 0.08, \\ h &= 15'33.70 \pm 1.0, \end{aligned}$$

The monograph [2] contains detailed investigations of the parameters of lunar physical libration. On the basis of extensive observational data, processed with allowance for the influence of phase for the crater Moesting A, Bel'kovich obtained the following values:

$$\begin{aligned} \lambda &= -5^{\circ}09'20'' \pm 9'', & I &= 1^{\circ}32'00'' \pm 14'', \\ \beta &= -3^{\circ}10'41'' \pm 7'', & f &= 0.67 \pm 0.03, \\ h &= 15'32.88 \pm 0.73, \end{aligned}$$

He established an important circumstance: an ambiguity in determining f . /42 Assuming that the shortcoming in the classical method for processing observations is in the computation process (computations are made in two stages), Bel'kovich (and simultaneously the Polish astronomer Koziel) undertook a search for new methods for reducing lunar observations [2]. He proposed that corrections to the initial values be determined by writing conditional equations for a series of fixed values of the parameter f in a broad range, but with a small interval (for f_0). For each f_0 value the method involves computation of $[vv]$ and finding a f_0 value for which $[vv]$ becomes minimum.

Interesting studies were made by Spenser-Jones and Jeffreys. They determined the values of the parameters of lunar physical libration from observations (meridian observations and lunar star occultations) of the rate of secular motion of the lunar orbit perigee and node. According to the formulas derived by Brown and De Sitter, the difference between the observed and computed values of the perigee and node represent the influence of both the sun and the figures of the earth and moon. Proceeding on this basis, Spenser-Jones [92] obtained the value $f = 0.68 \pm 0.09$. Jeffreys [77] first obtained $f = 0.97 \pm 0.13$, but later $f = 0.84 \pm 0.08$ [78].

The Polish astronomer Koziel made a significant contribution to study of lunar rotation. In 1948-1949 he published [79], in which a new method was used for the first time in computing the parameters of lunar physical libration, proposed by T. A. Banachiewicz. In this method conditional equations are written for each measurement of the lunar disk. The most important contribution of Koziel is that he attentively examined the classical theory of lunar physical libration in longitude and obtained more precise

numerical values of the coefficients when computing physical libration in longitude. In addition to determining the usually assumed unknowns, he determined the A. A. Yakovkin libration effect coefficient. Koziel concluded that the libration effect does not show up when it is determined jointly with other unknowns simultaneously for the western and eastern limbs.

The rotational parameters were determined both with and without allowance for corrections for lunar limb zone relief. The result obtained by Koziel was the same in both cases. True, with allowance for relief corrections, all the sought-for values were obtained with a smaller rms errors. In his investigations Koziel used the Hartwig series; as a result of the processing he obtained:

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when $f_0 = 0.50$, with allowance for relief

$$\begin{aligned}\lambda &= -5^{\circ}11'16'' \pm 13'', & I &= 1^{\circ}31'10'' \pm 22'', \\ \beta &= -3^{\circ}09'56'' \pm 18'', & f &= 0.60 \pm 0.055, \\ h &= 15'33''.81 \pm 0''.58,\end{aligned}$$

when $f_0 = 0.73$

Without allowance for relief

With allowance for relief

$$\begin{aligned}\lambda &= -5^{\circ}12'00'' \pm 17'', \\ \beta &= -3^{\circ}10'34'' \pm 26'', \\ h &= 15'32''.56 \pm 0''.84, \\ I &= 1^{\circ}31'08'' \pm 34'', \\ f &= 0.715 \pm 0.083;\end{aligned}$$

$$\begin{aligned}\lambda &= -5^{\circ}11'50'' \pm 12'', \\ \beta &= -3^{\circ}10'27'' \pm 17'', \\ h &= 15'32''.88 \pm 0''.56, \\ I &= 1^{\circ}31'36'' \pm 23'', \\ f &= 0.71 \pm 0.05.\end{aligned}$$

Beginning in 1938, and to the present time, lunar observations with a heliometer have been made without interruption by A. A. Nefed'yev. The first series, containing 143 observations of the crater Moesting A, was processed by him and published in 1951 [24]. Among the total number of observations, 79 were made prior to the full moon, three during the full moon, and 61 after the full moon. The processing was by the Bessel-Wichmann method. Corrections for lunar limb irregularities were taken from the Hayn maps (1914). Since these maps do not cover the entire lunar limb zone for different librations, it was necessary to extrapolate two individual points.

The normal equations, written for longitude and latitude, were initially solved separately, but then the latitude equations, multiplied by a factor greater than unity (since latitude equations are always more precise than longitude equations), were combined with the longitude equations. The solution of these equations gave the following values of the sought-for parameters (for $I_0 = 1^{\circ}32'20''$ and $f_0 = 0.73$):

$$\begin{aligned}\lambda &= -5^{\circ}10'13'' \pm 14'', & I &= 1^{\circ}32'04'' \pm 15'', \\ \beta &= -3^{\circ}11'46'' \pm 9'', & f &= 0.65 \pm 0.046, \\ h &= 15'33''90 \pm 0''45,\end{aligned}$$

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It therefore can be seen that the f value was close to the critical value. Accordingly, A. A. Nefed'yev decided to make a second approximation, in which, stipulating a series of f values systems of equations with four unknowns ($d\lambda$, $d\beta$, dH , dI) are written. Solving them by the least squares method, that value is selected from the series of tested f values at which the sum of the squares of the deviations is minimum.

After obtaining two most probable values, close to 0.60 and 0.71, A. A. Nefed'yev assumed as his initial value $f = 0.60$. Processing observations by the mentioned method, after a double solution (with $f_0 = 0.60$), the author obtained the following values of the sought-for parameters:

$$\begin{aligned}\lambda &= -5^{\circ}10'18'' \pm 15'', & I &= 1^{\circ}32'10'' \pm 16'', \\ \beta &= -3^{\circ}11'46'' \pm 10'', & f &= 0.57 \pm 0.041, \\ h &= 15'34''11 \pm 0''45,\end{aligned}$$

In addition, in [24] the author made graphic investigations of the effect of the phase of the crater Moesting A on observations. No such effect was discovered.

Thus, by 1951 all series of Kazan' heliometric observations had been processed, except for the first series, that made by Krasnov. In 1950 A. A. Nefed'yev undertook the processing of this series and in 1952 published the preliminary [25], and in 1955, the final and detailed results [26]. The processing was done in a selenocentric ecliptic coordinate system, i.e., the same as used in processing all the preceding heliometric series of observations. The following values were obtained:

$$\begin{aligned}\lambda &= -5^{\circ}11'08'' \pm 15'', & I &= 1^{\circ}32'10'' \pm 18'', \\ \beta &= -3^{\circ}11'38'' \pm 11'', & f &= 0.63 \pm 0.05, \\ h &= 15'34.43 \pm 0.45,\end{aligned}$$

This last study, to all intents and purposes, was processed by the Bessel-Wichmann method. If it is taken into account that the beginning of the first series of observations dates back to 1895, they have now been continued for more than 70 years. The observations are unique, the only ones in the world, and are of great scientific value, particularly since in duration they constitute the longest series of observations made on the same instruments and by the same method. /45

A study by Sh. T. Khabibullin [36] was published in 1955; it is also devoted to a determination of the parameters of lunar physical libration. In this study the author solves the problem of determining the parameter of physical libration f from the amplitude of the harmonic $a_3 \sin g'$ since the latter agrees well with the change in the f value. Accordingly, the idea arose to find the function a^3 (not having a discontinuity), and use this in determining the f value. Schrutka-Rechtenshtamm made an unambiguous determination of the f value by the approximation method [90]. In the computations he used the formula

$$K = (K)_{f_0} + \left(\frac{aK}{df} \right)_{f_0} df_0,$$

where K is the amplitude of the wave 2ω . Since the formula contains the differential correction df_0 , two f values were obtained.

A study by A. A. Yakovkin [50] was also devoted to determination of the f value from the wave $a_3 \sin g'$. In this study an attempt was made to apply harmonic analysis for determining the principal term in the series τ , expressing physical libration in longitude. The already processed Kazan' heliometric series of observations (A. V. Krasnov, I. A. Banachewicz, A. A. Yakovkin, I. V. Bel'kovich and A. A. Nefed'yev), made over a period of 50 years, were used in finding $\tau = l - l_1 = \Delta l$; these values were broken down

into 12 groups. From a solution by the least squares method, a value $\alpha = 76'' \pm 9''$ was obtained; this corresponds to $f = 0.69 \pm 0.02$

A joint determination of the parameters of forced and free lunar libration has been made in a study at the Institute of Theoretical Astronomy by M. V. Fridlyand [34]. These investigations were based on two series of heliometric observations separated by a long time interval, the Hartwig series (1884-1885) and the Nefed'yev series (1938-1945). These series cover a time interval of more than 60 years; this is very important for drawing conclusions concerning free libration. In order to simplify the computations, only free libration in longitude was determined. The f value was not determined directly, but was stipulated by a series of values: 0.58, 0.60, 0.62, 0.71, 0.73, 0.75. /46

Thus, for each indicated f value the same initial values λ_0 , β , h_0 , I_0 and R_0 were used. Only six systems of conditional equations were derived for the number of f values used. The solution of each of these separately yielded the corrections to the assumed values of the parameters of lunar physical libration and the free libration value. M. V. Fridlyand feels that the most probable f value will be that for which the sum of the squares of the deviations $[pvv]$ attains a minimum. In an analysis of the accumulated data it was found that this occurs for two f values: 0.62 and 0.73.

With respect to the amplitude of free libration in longitude, it was found to have the same order of magnitude as the amplitude of the maximum forced libration waves. It was therefore concluded that determination of the parameters of lunar physical libration must be done jointly with determination of free libration. The author stipulates no preference for either of the determined f values.

As a continuation of this work, M. V. Fridlyand in 1961 published a second study [35] devoted to a case when the f value is close to the critical value, i.e., when the amplitude of the term with the argument 2ω increases to infinity. Here the author examines the known differential equation of physical libration in longitude in a somewhat modified form. When $f = 0.662$, the period of free libration becomes equal to the period of a wave with the

argument 2ω , but if it is not equal to 0.662, but close to it, integration becomes ordinary, but the periods of free and forced libration do not differ over a period of hundreds of years.

Using these same Hartwig and Nefed'yev series, the amplitude of the forced libration for $f = 0.660$ was computed. As a result, the author feels that the amplitudes of the free and forced libration waves are limited in magnitude, i.e., the differential equation of physical libration in longitude has a solution near the critical point for the f parameter.

In 1958 a study was published by Sh. T. Khabibullin [37]; it represents a considerable contribution to study of lunar rotation. In 1949, at the Engel'gardt Astronomical Observatory, I. V. Bel'kovich constructed a photographic horizontal telescope with a long focal length ($F = 8$ m, $D = 12$ cm). This instrument was intended for lunar research: study of lunar limb zone relief, selenographic investigations, and tests of the photographic method for determining the parameters of lunar physical libration. A rather detailed description of the instrument is given in [37]. /47

This instrument was used in finding the parameters of lunar physical libration by the photographic method. The null point of the position circle is determined from the diurnal motion of the moon (printing of three images on a single plate with the telescope remaining in a fixed position).

Naturally, we feel that this method is considerably inferior to the method of photographing the moon among the stars. It may be that for the task which the author set himself, precise orientation is not essential, but for other problems for which lunar photographs could be used, this method is not adequately precise. Stars from the constellations Pleiades or Coma Berenices were printed on the plate for determining plate scale.

The observations were initiated in September 1949 and continued through May 1952. Only 40 lunar photographs were included in the processing. The plate measurements were made on a Repsold measuring instrument at the Shternberg Astronomical Institute. Sh. T. Khabibullin was the first to process observations in the Soviet Union by the Cracovian method. After describing the Bessel-Wichmann method in detail, the author points out its

shortcomings. Sh. T. Khabibullin formulates an interesting problem: instead of the direct determination of the value for the f parameter, compute some function of f which is determined without ambiguity.

It is known from theory that τ , ρ and $I\sigma$ can be represented in the form of a series

$$\begin{aligned}\tau &= (\tau)_{c0} + a_1 \sin g + a_2 \sin 2g + a_3 \sin g' + a_4 \sin 2\omega + \dots, \\ g &= (g)_{c0} + b_1 \cos g + b_2 \cos (g + 2\omega) + b_3 \cos (2g + 2\omega), \\ I\sigma &= I(\sigma)_{c0} + c_1 \sin g + c_2 \sin (g + 2\omega) + c_3 \sin (2g + 2\omega).\end{aligned}$$

It can be seen from the tables of amplitude a_i , libration in longitude (τ) and wave amplitudes ρ and $I\sigma$, given in the mentioned study, that libration in longitude is primarily dependent on the a_3 wave and that the other waves change little with a change in f , except for a_4 , which becomes greater with the approach of f to a critical value.

Sh. T. Khabibullin proposes that determination of the unknown Δf be replaced by determination of the amplitude of the main waves $a_3 \sin g'$ and $a_4 \sin 2\omega$ in physical libration in longitude. For this purpose he gives the corresponding equations for observations in Cracovian form, as follows

$$\left\{ \begin{array}{c} a_3 \\ a_4 \\ \Delta H \\ \Delta I \\ \Delta \lambda \\ \Delta \beta \\ \Delta R \\ 1 \end{array} \right\} \left\{ \begin{array}{c} a \\ b \\ c \\ l \end{array} \right\} \left\{ \begin{array}{cccccccc} A & B & C & D & E & F & O & L \\ A' & B' & C' & D' & E' & F' & O' & L' \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{R'_a}{R_0} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{array} \right\} = 0,$$

where $i = 1, 2, \dots, r$, $j = 1, 2, \dots, s$, $n = \sum_{j=1}^s r_j$.

In addition, a_3 and a_4 are related by the dependence

$$2.7442 a_3 - 83.6607 a_4 \pm 100 a_3 a_4 = 0.$$

The f value is determined using the formula

$$f = \frac{\beta - \gamma}{\beta^2}, \quad \gamma = \frac{\left(\frac{dh}{dt}\right)a}{0.9856 M'_a - M'H}.$$

By solving 1,168 equations, corresponding to the number of measured points on the lunar limb, the author obtained:

$$\begin{aligned}\lambda &= -5^{\circ}10'08'' \pm 12'', & I &= 1^{\circ}33'06'' \pm 11'', \\ \beta &= -3^{\circ}10'37'' \pm 8'', & f &= 0.50 \pm 0.05, \\ h &= 15'33''7 \pm 0''7, \\ R_0 &= 15'32''59 \pm 0''03,\end{aligned}$$

The f value was small. The author attributes this to the fact that the results were obtained using the Koziel model. Accordingly, in order to compare the accuracy of the two methods, observations were processed using the Bessel-Wichmann method; this gave

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$$\begin{aligned}\lambda &= -5^{\circ}09'52'' \pm 31'', & I &= 1^{\circ}32'58'' \pm 22'', \\ \beta &= -3^{\circ}11'06'' \pm 13'', & f &= 0.58 \pm 0.08, \\ h &= 15'34''3 \pm 0''9,\end{aligned}$$

As we see, the results are different, particularly with respect to the crater latitude and the f value. With respect to observation errors, in the second case the mean square errors were considerably increased.

Sh. T. Khabibullin used his method in processing heliometric series of observations made at Kazan' and obtained

Theories	f
Krasnova	0.58 ± 0.08
Mikhaylovskiy	0.61 ± 0.09
Banachiewicz	0.60 ± 0.03
Yakovkin	0.63 ± 0.01
Bel'kovich	0.61 ± 0.03
Nefed'yev	0.59 ± 0.04

On the average, from these data $f = 0.60 \pm 0.02$. Sh. T. Khabibullin considers the probable value to be $f = 0.60$.

Observations similar to heliometric observations, were made in 1955 by V. K. Drofa [17] on the proposal of A. A. Yakovkin. He used a specially designed wire micrometer in measuring the coordinates (distances s and position angles p) of selected points uniformly situated on the illuminated lunar disk limb relative to the position of the crater Moesting A. In making these observations he used the A. A. Yakovkin eyepiece; this made it possible

to bring the images of the lunar limb and the crater into the field of view next to one another. All observations were reduced for irregularities of the lunar limb using the Weimer profiles. In computing the instrumental errors, V. K. Drofa made a study of the instrument and the position circle null point. In order to use the Weimer profiles, he proposed a spatial interpolation method using the collected data he computed: the correction for the lunar radius and the selenocentric coordinates of the crater Moesting A. The differences in the observed and ephemeris values were represented as functions of the corrections to coordinates, equatorial inclination I and f parameters. The longitude and latitude equations were combined; the latitude equations were multiplied by a factor of 2.45. The observations were made on a 10-second refractor. The following values were obtained

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$$\begin{aligned} \lambda &= -5^{\circ}08'17'' \pm 57'', & I &= 1^{\circ}27'56'' \pm 63'', \\ \beta &= -3^{\circ}09'16'' \pm 38'', & f &= 0.72 \pm 0.21, \\ h &= 15'28'' \pm 2''1, \end{aligned}$$

The value of the coefficient of the libration effect in radius was found to be considerable ($+0.115 \pm 0.027$, with a mean optical libration $\beta = +8^{\circ}$). In an analysis of the arrangement of observations relative to optical libration, it was assumed that the lunar radius changes nonlinearly with optical libration in latitude. Still another peculiarity of this study should be mentioned: the high mean square error of unit weight (two times greater than for heliometric observations). Naturally, this affected the accuracy of the results; the reason was evidently some aspect of the observation method. In addition, the Weimer lunar limb profiles cannot give a satisfactory accuracy.

The special double micrometer eyepiece, used for the first time by V. K. Drofa for lunar observations, can be adapted to ordinary refractors and replace heliometric observations by observations with refractors, whose advantage is a large magnification and image clarity. The heliometer does not give star images of a regular configuration because each half of the objective operates independently and the star diffraction pattern is not a circle, but a line.

In 1961 a study by Sh. T. Khabibullin [38] was published; it was devoted to a determination of the parameters of lunar physical libration from a processing of the Hartwig series (Dorpat) on the basis of data processed by Koziel. Instead of determining the f parameter, the amplitude of the largest waves in the physical libration in longitude $a_3 \sin g'$ was computed; then the a_3 value was used in computing f . The theoretical values $d\lambda$ and $d\beta$, given by Hayn, were expressed in the form of two waves, i.e., $a_3 \sin g'$ and Ω . In this procedure the a_3 amplitude is considered an unknown, whereas Ω is known (but it is necessary to stipulate the determined f value). The approximations method is used in determining the Ω and $d\beta$ values which exert little influence on determination of f . The material used in determining the parameters of lunar physical libration were the $s_0 - s_C$ values computed by Koziel. By solution of normal equations, after the second approximation, the following values of the unknown parameters were obtained: /51

$$\begin{aligned} \lambda &= -5^{\circ}11'03'' \pm 9'', & R_g &= 15'37.86 \pm 0.019, \\ \beta &= -3^{\circ}10'03'' \pm 8'', & I &= 1^{\circ}31.22 \pm 14'', \\ h &= 15'33.28 \pm 0.35, & f &= 0.60 \pm 0.019. \end{aligned}$$

With respect to the presence of a libration effect in the lunar radius, according to the investigations made in this study, this parameter was equal to zero.

The Hartwig series (1884-1885), consisting of 36 observations, was processed by Koziel. A. A. Nefed'yev processed his own heliometric observations [28], which he had earlier processed by the Bessel-Wichmann method, and by the Cracovian method as well. Taking into account that the fundamental constant is found to have two values, A. A. Nefed'yev computed the f value for the two initial values $f_0 = 0.50$ and $f_0 = 0.73$. Using the Banachiewicz successive approximations method, A. A. Nefed'yev obtained the constants of lunar physical libration for $f_0 = 0.73$:

$$\begin{aligned} \lambda &= -5^{\circ}10'05'' \pm 11'', & R_g &= 15'32.44 \pm 0.025, \\ \beta &= -3^{\circ}12'07'' \pm 8'', & I &= 1^{\circ}32'26'' \pm 14'', \\ h &= 15'36.14 \pm 0.42, & f &= 0.71 \pm 0.025. \end{aligned}$$

For an initial value $f_0 = 0.50$, after two approximations he obtained:

$$\begin{aligned}\lambda &= -5^{\circ}10'04'' \pm 11'', & R_L &= 15^{\circ}32'47'' \pm 0.02, \\ \beta &= -3^{\circ}12'10'' \pm 9'', & I &= 1^{\circ}33'24'' \pm 14'', \\ h &= 15^{\circ}36'04'' \pm 0.42, & f &= 0.63 \pm 0.03.\end{aligned}$$

It can therefore be seen that the values of the selenographic coordinates of the crater, as well as h , R_L and I are close, for f the difference was 0.08. This once again confirms that the Banachiewicz approximation method automatically gives an f value in that direction for which the initial f_0 value was taken. /52

If we compare the results with those obtained by the classical method employed earlier by the author, it immediately becomes clear that the accuracy in processing was appreciably increased by the Cracovian method. The h and I values differ considerably. The investigation reveals that the libration effect is not observed with joint processing of both limbs, but shows up with separate processing.

A study by K. S. Shakirov [41] was published in that same year; the author determined the parameters of lunar physical libration from meridian observations of the crater Moesting A made at the Greenwich Observatory during the years 1952-1954. K. S. Shakirov used the "method of determining the coordinates of the crater Moesting A relative to the center of lunar mass by tying the crater to the stars."

The basis for the processing was the Cracovian computation method. Eighty-nine meridian observations of the crater Moesting A over a two-year period were used in determining an unambiguous value of the f parameter. Processing of this series of observations gave the following values of the parameters (for $I_0 = 1^{\circ}32'20''$ and $f_0 = 0.73$):

$$\begin{aligned}\lambda &= -5^{\circ}09'46'' \pm 23'', & I &= 1^{\circ}30'54'' \pm 30'', \\ \beta &= -3^{\circ}12'44'' \pm 19'', & f &= 0.71 \pm 0.08, \\ h &= 15^{\circ}31'16'' \pm 0.8,\end{aligned}$$

The β , h and I values differ appreciably from the initial values employed, whereas λ and f were close to the initial values.

The study [41] is interesting because the author examined the problem of determining parameters without a tie-in to the lunar limbs. The assertion made by K. S. Shakirov that his method is free of systematic errors associated with the lunar limb when determining I and f is erroneous. He does not observe the limbs, but lunar meridian observations are related to the limbs and in the processing the lunar figure is assumed to be spherical. The errors arising from this in determining the elements of the lunar orbit result in ephemeris and other errors. The fact is that the lunar orbital elements were derived from the connection between stars and the lunar limb, and not a connection with the lunar center of mass, and in processing such observations the libration effect in the apparent lunar figure is not taken into account. Accordingly, the lunar orbital elements must be corrected for libration, and only after such reworking of meridian observations will the ephemeris for the lunar center of mass correspond to reality.

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The Polish astronomers I. Maslovski and I. Mitelski [83] processed the Hartwig series (1890-1922) and the Banachiewicz series with the introduction of corrections from the Hayn maps. The processing was done using a spherical model with the Koziel Cracovian method. The following values were obtained from each series:

Hartwig (Bamberg) Series

$$\begin{aligned}\lambda &= -5^{\circ}09'26'' \pm 5.2, \\ \beta &= -3^{\circ}11'00'' \pm 4.5, \\ h &= 15^{\circ}33'03'' \pm 0.187, \\ l &= 1^{\circ}31'58'' \pm 6.8, \\ f &= 0.627 \pm 0.012, \\ R_0 &= 15^{\circ}32'98'' \pm 0.012,\end{aligned}$$

Banachiewicz (Kazan') Series

$$\begin{aligned}\lambda &= -5^{\circ}10'05'' \pm 6.2, \\ \beta &= -3^{\circ}10'52'' \pm 6.3, \\ h &= 15^{\circ}31'82'' \pm 0.281, \\ l &= 1^{\circ}32'37'' \pm 10.0, \\ f &= 0.628 \pm 0.019, \\ R_0 &= 15^{\circ}32'83'' \pm 0.019.\end{aligned}$$

The initial f_0 value was assumed to be 0.60. The convergence of f values for both series was very good, but since the processing was done using Hayn's maps, based on a spherical lunar model, the fundamental errors in these maps are reflected in the results.

An interesting study was published in 1966 by S. G. Valeyev [4]. He processed the same Hartwig series, with the introduction of limb relief corrections from the Watts maps. The Sh. T. Khabibullin method, discrimination of the $a_3 \sin g'$ wave, was used for an unambiguous determination of the f parameter. The computations, made by the Cracovian method, gave results different from those obtained by I. Maslovski and I. Mitelski, as well as those obtained by Naumann, i.e.,

$$\begin{aligned} \lambda &= 15^\circ 32' 57'' \pm 0.014, & l &= 1^\circ 32' 30'' \pm 5.1, \\ \lambda &= -5^\circ 10' 50'' \pm 3.8, & f &= 0.78 \pm 0.014, \\ \mu &= -3^\circ 12' 01'' \pm 3.3, & R_d &= 15^\circ 32' 98'' \pm 0.009. \end{aligned}$$

The f value obtained by S. G. Valeyev was very close to the values now obtained using the "model B" and the position angles method.

In addition, Valeyev attempted to clarify the influence of the systematic errors on the Watts maps on the determined parameters. In this study he employed the Dorpat heliometric series earlier processed by Sh. T. Khabibullin; in this series he corrected the free terms for relief using the Watts maps. He obtained the very same results, thus indicating that the Watts maps in actuality do not differ from the Hayn maps. /54

It should be emphasized very clearly that during recent years active investigations have been made in the Soviet Union not only on making observations by a new method, but also in formulating significant theoretical works devoted to investigation of lunar rotation and motion. During recent years interesting investigations have been made at the Institute of Theoretical Astronomy (Leningrad) by S. G. Makover and M. D. Polanuyer. We will discuss these investigations briefly. S. G. Makover, in a study entitled "Some Problems in the Theory of Lunar Physical Libration" [22], writes that the expansions of the right-hand sides of the Euler equations in series, performed by Hayn and Koziel, are satisfactory only for the components ρ and $I\sigma$, having shortcomings for the component τ . He pointed out that "such an unsatisfactory form of the solution is probably associated with the peculiarities of the approximate integration method employed by Hayn and Koziel, and

the fact that we are not able to integrate the differential equation." Makover considers the integration method to be unsuitable, because in solving the differential equation for the τ function and reducing it to a form convenient for integration we neglect some "secondary" terms. He reduces the equations to a parametric form. True, Sh. T. Khabibullin [39, 40] points out that resonance solutions also remain in the case of parametric oscillations. Unfortunately, Makover's idea has not been applied in practical studies.

A study by M. D. Polanuyer [29, 30] was devoted to an examination of the theory of lunar physical libration. In deriving the parameters of lunar physical libration he integrates the dynamic and kinematic equations by the Charlier method in canonical form. In contrast to Johnson [76], he solves the nonlinear problem, integrating the canonical equations by the Linshtedt Poincaré asymptotic method. The solution was given in the form of purely trigonometric series. This theory was used by the author in processing very limited observational material, and we will not give the results. /55

Sh. T. Khabibullin, at the Kazan' City Astronomical Observatory, recently has prepared studies [39, 40] devoted to an investigation of nonlinear oscillations in lunar rotation. The earth is assumed to be a point and the moon a triaxial ellipsoid. Where $f = 0.662$ the linear theory gives a discontinuity, whereas in nonlinear theory, in the case of resonance ($f = 0.662$), lunar rotation will be stable. The Euler dynamic equations are written in aircraft angles of roll, pitch and yaw. Terms with minimum coefficients are taken into account. The system of dynamic equations is reduced to a system of quasilinear equations. The solutions are obtained in the neighborhood of the zero value of a small parameter. The stability of almost periodic solutions is investigated with the presence of resistance in lunar rotation. It is shown in this study that free libration does not occur in nonlinear theory. The numerical representation of nonlinear theory is based on the lunar tables compiled by Brown. The advantage of aircraft angles over Eulerian angles is that they make it possible to obtain two differential equations in a form convenient for integration.

As we see, numerous studies devoted to determining the parameters of lunar physical libration were made on the assumption that the moon is a sphere. In the processing it was assumed that the most probable circles have a common center on different dates and that their radii are identical, with the circles supposedly pertaining to the same sphere. However, this does not correspond to reality. In 1934 A. A. Yakovkin (and others at later dates) demonstrated that the apparent lunar radius changes in dependence on optical libration in latitude. This dependence is expressed by the formula

$$h = h_0 + 0.05\beta_0,$$

where β_0 is libration in latitude, expressed in degrees.

Change in lunar radius undoubtedly exerts an effect on determinations in the parameters of lunar physical libration.

A. A. Yakovkin was the first to take this influence into account in the processing of the Banachiewicz theories and his own series of heliometric observations; as a result he obtained [43]:

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$$\begin{aligned}\lambda &= -5^{\circ}11'13'' \pm 14'', & I &= 1^{\circ}33'48'' \pm 17'', \\ \beta &= -3^{\circ}13'11'' \pm 11'', & f &= 0.82 \pm 0.03, \\ h &= 15'34.52 \pm 0.5,\end{aligned}$$

In comparison with the initial values, there was a particularly noticeable increase in I (when $I_0 = 1^{\circ}32'20''$) and f (when $f_0 = 0.75$). This result for I could be foreseen, since in determining the libration effect a term proportional to β_0 is added to the right-hand side of the conditional equations for latitude. This new term correlates with the coefficients on dI on the left-hand side of the conditional equations. The decrease in the selenographic latitude of the crater can be attributed to the fact that the center of the lunar figure is situated to the south of the center of mass.

Model "B" was used in this processing. Later Yakovkin proposed a different model, Model "C", in which the asymmetry of the lunar disk is taken into account not only relative to the lunar equator, but also relative to the central meridian. It obviously is a better representation of the true lunar

figure. Lunar asymmetry is manifested in the fact that the libration effect in radius is different for the lunar eastern and western hemispheres. In model "C" the semimajor axis of the ellipse (for the southern hemisphere) forms a definite angle γ with the lunar axis of rotation.

It was interesting to process other series of observations by the same method. In 1960 the author of this monograph recomputed the Hartwig observations processed earlier by Naumann [42]. It was important to make a proper selection of the mathematical curve which while smoothing small lunar limb irregularities would express the systematic deviation from a circle under the condition that the stipulated point associated with this curve also coincides in the projection with the lunar center of mass. Such a curve was proposed by A. A. Yakovkin [53]. We proposed a different curve which we feel better emphasizes the local nature of the elevation in the lunar southern hemisphere [8].

In order to use the Hayn maps for obtaining radius vector corrections for relief, for relating it to the surface of a baricentric sphere, the elevation of the zero Hayn level above the surface of the baricentric sphere at the particular point must be added to the elevation H taken from the map. Formulas were written and tables compiled which give the elevation of the Hayn zero (reference) level above the baricentric sphere for points on the lunar profile. The basis for the computations was the equation for the curve

$$r = r_0 + a \cos^4 \varphi.$$

Here the fourth power of the cosine was used in order to reflect the local nature of the elevation in the southern hemisphere. Taking into account the values of the constants of physical libration for which the corrections were found, the following values of the parameters were found for $\gamma = 20^\circ$

$$\begin{aligned} A &= -5^\circ 10' 57'' \pm 13'', & I &= 1^\circ 32' 40'' \pm 15'', \\ B &= -3^\circ 12' 43'' \pm 10'', & f &= 0.82 \pm 0.029, \\ h &= 15' 32'' 21 \pm 0'' 52, \end{aligned}$$

The f value (0.82) was considerably greater than that found by Naumann (0.71). The value $f = 0.82$ agrees well with the conclusions drawn by Jeffreys, obtained

from a study of lunar orbital motion. This shows that allowance for the libration effect considerably changes the f parameter in the direction of an increase. However, we feel that taking into account the investigations of recent years we must assume that the γ angle is not 20° , but approximately 35° , which corresponds better to the true lunar figure.

A study by D. P. Duma was made at the Main Astronomical Observatory of the Ukrainian Academy of Sciences [20]; it was devoted to determination of the parameters of the lunar physical libration from heliometric observations made by A. V. Krasnov with major relief taken into account by the use of model "C". Processing gave the following values:

$$\begin{aligned}\lambda &= -5^\circ 11' 39'' \pm 15'', & I &= 1^\circ 32' 55'' \pm 23'', \\ \beta &= -3^\circ 13' 06'' \pm 9'', & f &= 0.75 \pm 0.06, \\ h &= 15' 33'' 11 \pm 0.36,\end{aligned}$$

Comparing the processing of this same series by Nefed'yev ($f = 0.63$), we see that this result corresponds to a libration effect in radius

$$\bar{r} = r_0 + 0.05\beta_0.$$

In general, the Yakovkin model "C" substantially changes the parameters of lunar physical libration.

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In the usual method for determining the parameters of lunar physical libration, employed since the time of Bessel, it is necessary to measure distances from some crater situated near the center of the lunar figure to different limb points for different librations. Such observations have been made using a heliometer or the photographic method. A shortcoming of such observations is an uncertainty caused by an inadequate knowledge of the lunar limb profile.

In 1955 Watts made an interesting study at the Washington Observatory [94]. A large number of lunar photographs was obtained using an astrograph with a long focal length ($F = 12.2$ m, $D = 12.7$ cm) for compiling maps of lunar profiles. Watts used a series of these photographs in determining the inclination of the lunar equatorial plane (I) to the ecliptic. In determining the position angle P of some feature on the lunar limb, reckoned from the

northern end of the lunar axis, it is necessary to subtract the position angle C of the lunar axis, which is dependent on the inclination of the lunar equator to the ecliptic from the measured position angle π of this feature relative to the north point, i.e., $P = \pi - C$. This was done through special selection of two lunar photographs taken at different times but at close librations and with similar limb illumination. The measured position angles of the same features on the two photographs should differ only from the error in C and the elements of lunar physical libration (neglecting measurement errors). The measured position angles of the same disk feature on both photographs are used in writing conditional equations. A total of 78 such equations was written for different combinations of 88 plates. Solution of these equations gave the value

$$I = 1^{\circ}33'50''.$$

We therefore see that the inclination of the lunar equatorial plane to the ecliptic is greater than given by Hayn by $1'30''$. This method is unquestionably interesting but the observed feature on the lunar limb is obviously difficult to register precisely. Accordingly, this method may contain considerable random measurement errors.

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In 1965 the equations of forced physical libration were solved numerically [63]. The solution of differential equations defining the forced lunar physical libration is given. The equations were integrated on electronic computers by the successive approximations method with an accuracy to 0.1 . The author determined perturbations relative to Cassini's laws for preset values of the principal moments of lunar inertia and some combinations of numerical values of the parameters β and γ . It was shown in this study that the influence of perturbations of the planets on libration in longitude can be neglected because this influence is small.

The paper "Motions of the Lunar Perigee and Node and the Distribution of Lunar Mass" was published in 1965 [64]. The gravitational effect of the sun and planets, the relativistic corrections to Newton's laws of gravitation, as well as the asphericity of the earth and moon, cause secular motions of the lunar perigee and node. Brown's theory with modern values of the parameters

shows that if the perturbing factors are taken into account (except for lunar asphericity), the difference between the observed and computed value for a century will give - 3".1 in perigee motion and - 27" in node motion. The numerical value of the influence of lunar asphericity is characterized by the parameters

$$f = (C' - B') / (C' - A') \text{ and } g' = 3C' / 2MB'^2,$$

where A , B and C are the lunar moments of inertia, M is lunar mass, B' is the lunar equatorial radius. The parameter g' , characterizing the radial distribution of lunar mass, is 0.6 for a homogeneous spherical moon and 1.00 for a full spherical layer.

If the mentioned differences arise as a result of lunar figure asphericity, the values $f = 0.638$, $g' = 0.965$ are obtained for the parameters. This indicates that the density of the lunar layers increases with approach to the surface. The value $g' = 0.6$ leads to an inexplicable discrepancy between the observed and computed values for node motion.

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In May 1966 Arthur published a study entitled "Method for Determining the Constants of Lunar Rotation from Measurement of Scale and Oriented Photographs" [58]. The author of this study notes that the heliometer method belongs to the last century and has its shortcomings. The method proposed by Arthur is based on the use of photographs; these make it possible to make a large number of measurements between well-determined features on the lunar surface without a tie-in to the limb. On the photographs the orientation and scale of the lunar image must be known.

The observation equations have the form:

$$\begin{aligned} \frac{\partial x}{\partial \beta} \delta \beta + \frac{\partial x}{\partial \gamma} \delta \gamma + \frac{\partial x}{\partial u} \delta u + \frac{\partial x}{\partial v} \delta v + \frac{\partial x}{\partial w} \delta w &= x_0 - x_c, \\ \frac{\partial y}{\partial \beta} \delta \beta + \frac{\partial y}{\partial \gamma} \delta \gamma + \frac{\partial y}{\partial u} \delta u + \frac{\partial y}{\partial v} \delta v + \frac{\partial y}{\partial w} \delta w &= y_0 - y_c, \end{aligned}$$

where u , v , w are the selenodetic coordinates with their origin at the crater Moesting A, β and γ are known values, x and y are the photographic coordinates with their origin at the crater Moesting A and with a y -axis parallel to the

hour circle through the center of the disk. In conclusion, the author points out that the described method is evidently the first attempt at departure from the limited classical selenodesy. However, we cannot agree with this, since this method is a duplication of the method proposed by A. A. Yakovkin. Moreover, the A. A. Yakovkin method has already been tested [10, 11, 12, 13], and the present work is an important application of the A. A. Yakovkin method in practice.

The Period of Free Libration

The period of free libration is a function of the lunar moments of inertia; it can be computed for a definite f . The amplitude and phase are determined as arbitrary constants for integration of equations derived from observations. It follows from theory that there are three free libration waves in longitude with periods of one month, three years, and 24 years. We know virtually nothing definite concerning other free libration waves.

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The "Astronomical Almanac of the USSR", as well as foreign almanacs, contain ephemerides of lunar physical libration computed on the assumption that free libration does not exist. At the present time it is difficult to say how legitimate this assumption is.

The first attempt at determining free libration in longitude was made by Wichmann in [97]; by the processing of 50 heliometric observations of the crater Moesting A he obtained the amplitudes and phases of two waves, as follows:

$$\begin{aligned} a \sin A &= -35^{\circ}3' \pm 88^{\circ}5', & b \sin B &= +17^{\circ}3' \pm 224^{\circ}7', \\ a \cos A &= +174^{\circ}3' \pm 113^{\circ}0', & b \cos B &= +3^{\circ}0' \pm 114^{\circ}0'. \end{aligned}$$

The errors in these determinations are so great that the results can be considered invalid. In his computations Wichmann allowed errors which Hartwig later discovered in processing this series. He obtained free libration values close to those obtained by Wichmann. In processing his Strassburg series of heliometric observations, Hartwig [67] obtained the following values:

$$\begin{aligned} a \sin A &= -62^{\circ}3' \pm 22^{\circ}8', & b \sin B &= +294^{\circ}6' \pm 131^{\circ}7', \\ a \cos A &= -0^{\circ}9' \pm 35^{\circ}1', & b \cos B &= +56^{\circ}4' \pm 47^{\circ}6'. \end{aligned}$$

We see from the determinations made by Wichmann and Hartwig that the phase and amplitude values are sharply different. Hartwig's last observations were reprocessed by Franz [65]; he once again obtained different results, and also with large errors:

$$\begin{aligned} a \sin A &= + 57''8 \pm 26''6, & b \sin B &= + 24''4 \pm 34''2, \\ a \cos A &= - 31''1 \pm 51''7, & b \cos B &= + 93''8 \pm 44''6. \end{aligned}$$

The great discrepancy in the results led Franz to the conclusion that free libration determinations have a low reliability. Accordingly, no one made free libration determinations for a long time.

Free libration in longitude was found by A. A. Yakovkin from the heliometric observations made by Banachiewicz (1910-1915) and his own observations (1916-1931). In both cases free libration was determined from an analysis of the residual errors in determining the parameters of physical libration. A. A. Yakovkin, in explaining this method, points out that the period of free libration is dependent on the same unknown γ (or f) as the coefficients of forced libration.

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In order to determine free libration from the Banachiewicz series [42] it was assumed that $f = 0.74$; hence $\gamma = 1 - 0.74$; $\beta = 0.0001635$. As a result of an analysis of the remaining deviations v , in the conditional equation for longitude (with a period of 3.4 years) the arbitrary libration in longitude was found to be

$$\begin{aligned} \tau_{\text{free}} &= 35'' \sin [121^\circ + 0.288 (t - 241200)], \\ &\quad \pm 13'' \quad \pm 21^\circ \end{aligned}$$

This result is virtually the first reliable value for lunar free libration.

A. A. Yakovkin obtained a second free libration value from his own heliometric observations [45]. In determining the period he used the value $f = 0.72$, taken from the series of Hayn, Banachiewicz, Yakovkin and Hartwig. This f value corresponds to $\beta = 0.0006289$, and therefore to a period $P = 1,203$ days. The observations cover 4.5 free libration periods. In determining the free libration parameters (α and A) A. A. Yakovkin used harmonic analysis. In this work the deviations v in the conditional equations

were broken down into 100-day groups (corresponding to 1/12 of a period); then, combining into groups with identical phases, the following values were obtained:

$$\tau_{\text{free}} = + 31'' \sin \left[174^\circ + (t - 2421000) 0^\circ 30' \right] \pm 12'' \pm 20^\circ$$

Applying the A. A. Yakovkin method, in 1949 I. V. Bel'kovich, by processing the series observed by Hartwig [(1890-1915), Banachiewicz (1910-1915) Yakovkin (1916-1931), and his own observations (1932-1942), a total of 582 observations, assuming $f = 0.71$, $I = 1^\circ 31' 00''$, and hence $\beta = 0.00062899$ and $P = 1,184$ days, obtained the following values

$$\begin{aligned} \tau_{\text{free}} &= + 11'' \sin \left[95^\circ + 0^\circ 30' 41'' (t - 2412000) \right], \\ \tau_{\text{free}} &= + 40'' \sin \left[219^\circ + 0^\circ 30' 41'' (t - 2412000) \right], \\ \tau_{\text{free}} &= + 31'' \sin \left[310^\circ + 0^\circ 30' 41'' (t - 2412000) \right], \\ \tau_{\text{free}} &= + 13'' \sin \left[59^\circ + 0^\circ 30' 41'' (t - 2412000) \right]. \end{aligned}$$

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After analyzing these results, I. V. Bel'kovich concludes that they are not entirely reliable. He feels that if free libration does exist, its value falls beyond the limits of observation accuracy. According to his computations, the amplitude of free libration is $\alpha = 4'' \pm 5''$. A. A. Yakovkin, also adding the Nefed'yev series to these series, obtained $\alpha = 52'' \pm 10''$, $A = 208^\circ \pm 7^\circ$.

Sh. T. Khabibullin notes that in one of these results there is an error because virtually the same observations cannot give such different results. Khabibullin attributes this discrepancy to the fact that with exclusion of forced libration in longitude of the crater Moesting A, Yakovkin used in computing τ the value $f = 0.65$; this value is close to the critical value (0.662) and does not correspond to observations. He feels that it was necessary to use a value 0.60. In the difference

$$\tau_{f=0.60} - \tau_{f=0.65} = 13'' \sin g' + 61'' \sin 2\omega,$$

the residual wave 2ω has a period of 1,095 days, close to the period accepted by Yakovkin (1,085). Accordingly, the determined amplitude $\alpha = 52''$ is the amplitude of the residual difference $\tau_{f=0.60} - \tau_{f=0.65}$ and not the free difference. Khabibullin therefore concludes that the results obtained by Yakovkin and Bel'kovich are an indirect confirmation that $f = 0.60$.

In 1951 A. A. Nefed'yev [24], by processing his own observations (1938-1945), obtained a free libration value with its period for $f = 0.71$:

$$\tau_{\text{free}} = 21'' \sin [259^\circ + 0.02988 (t - 2411458)] + 32'' \sin [268^\circ + 0.01641 (t - 2411458)] .$$

In 1959 M. V. Fridlyand [34], from a joint processing of the two Hartwig and Nefed'yev series, separated by an interval of 61 years, found α and A values corresponding to different f values.

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For f	α	A
0.58	$38'' \pm 15''$	$31^\circ \pm 18^\circ$
0.60	28 ± 15	6 ± 28
0.62	38 ± 16	340 ± 19
0.71	10 ± 15	253 ± 80
0.73	48 ± 16	270 ± 17
0.75	33 ± 14	208 ± 24

In all the mentioned determinations, the libration effect in radius was not taken into account; naturally, this must be reflected in the results.

S. G. Makover [22], after analyzing all the results obtained up to the present time, concludes that "the amplitudes of all three free libration waves are exceedingly small and cannot be determined in the processing of observations." On the other hand, in setting forth his own opinion, he proposes:

a) to compute the expansion coefficients for the component τ of physical libration by a new method, i.e., by integrating differential equation (26); in his text this equation is of the Hill type;

b) by using these expansions, to process some series for a new determination of the constants of physical libration; at the same time to use a strictly unambiguous method for finding a new value of the f parameter; to endeavor to demonstrate that the hypothesis that the amplitude of all free libration waves are equal to zero does not contradict observations. Thus, S. G. Makover shares the opinion concerning absence of free libration.

On the other hand, A. A. Yakovkin feels that lunar free libration evidently does exist, but for the time being the problem of its characteristics cannot be considered finally solved.

Sh. T. Khabibullin writes on this problem in [40] that the use of nonlinear theory makes possible a correct conclusion concerning so-called free libration. In addition, he feels that the coefficients of free libration (α and A) have been considered constant, but in actuality variations in lunar rotation are nonlinear; therefore, the α and A coefficients must be regarded as new variables. /65

We see that the problem of investigating free libration has not yet been finally solved. There is no clear idea concerning the magnitude or even the very existence of free libration.

A great number of studies has been devoted to determination of the parameters of lunar physical libration on the basis of heliometric and photographic observations. However, all observations have been related to the apparent lunar limb, whose configuration varies with optical libration. In almost all studies the final results would be true if the centers of the circles determined for different optical librations coincided with the center of lunar mass and had the same radius. Processing of observations by the Koziel method does not eliminate this fundamental shortcoming. Accordingly, other methods for solving this problem must be found.

In 1961 A. A. Yakovkin proposed a method for determining the parameters of lunar physical libration which was free from the mentioned shortcomings [57]. He proposes determination of the position of the crater Moesting A (or some other crater situated near the center of the lunar figure) relative to the center of lunar mass which would not be dependent on the figure of the

lunar limb zone. Two methods are proposed: 1) measurement of distance from the crater Moesting A to small craters situated near the lunar disk limb, 2) measurement of position angles of the directions between the crater Moesting A and a limb crater. Preference is given to position angle observations.

This method was used by the author of this monograph [9] for the first time using photographic materials obtained with the refractor at the Astronomical Observatory of Kiev State University ($D = 200$ mm, $F = 4.3$ m) by A. K. Osipov and a small number of photographs obtained at the Main Astronomical Observatory of the Ukrainian SSR ($D = 400$ mm, $F = 5.5$ m) during the years 1950-1959. This method will be discussed in greater detail later and here we will give only the results obtained from processing this photographic material for the values $I_0 = 1^\circ 32' 20''$ and $f_0 = 0.73$ (see Table 5).

TABLE 5

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Series of Observations	I	Weight	f	Mean Error in one equation	No. of equations
First series	$1^\circ 33' 14'' \pm 29''$	55.42	0.87 ± 0.11	± 3.6	114
Second series	$1^\circ 33' 23'' \pm 17''$	71.98	0.91 ± 0.14	± 2.4	112
Combined series	$1^\circ 33' 19'' \pm 16''$	127.40	0.89 ± 0.13	± 3.4	126

Since September 1962 specialists at the Main Astronomical Observatory of the Ukrainian SSR have been making visual observations of the moon with a refractor ($F = 3$ m, $D = 200$ mm); this instrument has an ordinary wire micrometer with a position circle having a reading accuracy of $1'$. We feel that visual observations have advantages over photographic observations. On the basis of a processing of a series of 50 visual observations of position angles of pairs of craters during the period 13 September 1962 through 12 August 1963 we have [10]:

$$I = 1^\circ 33' 10'' \pm 11'',$$

$$f = 0.84 \pm 0.08.$$

The mean square error in one equation with a weight of one is $\epsilon = \pm 2.3$.

It is known that the earlier determined f values break down into two groups, separated by the critical value $f = 0.662$. The results of one group is closer to 0.60, whereas the results in the other group are closer to 0.73. Bel'kovich and Koziel mentioned the duality of the solution, depending on from what side of the critical value the initial f_0 value was taken. The sought-for f value is obtained on that side from which the f_0 value was taken. However, there are cases when the sought-for value is obtained on the other side of the critical value. Accordingly, it was necessary to compute the observational data by the position angles method for $f_0 = 0.60$. The processing gave the following values:

$$\begin{aligned} I &= 1^{\circ}32'34'' \pm 17'', \\ f &= 0.83 \pm 0.18. \end{aligned}$$

The mean errors in determining the values were large; this can be attributed to the small number of observations. The determined values (I and f) in this case must not be regarded as final. The determined f value is a confirmation that $f = 0.60$ cannot be used as an initial value because it differs considerably from the true value. In addition, this shows that the true f value is not always situated on that side of the critical value from which the approximate value was taken and it also shows that the true f value will be greater than the critical value. /67

CHAPTER III

THEORY OF LUNAR PHYSICAL LIBRATION

The theory of lunar physical libration has been set forth quite fully in many studies by both Soviet and foreign authors. Two types of bodies exist in the solar system which differ in their configuration. The sun and planets (except Mercury) are considered ellipsoids of revolution, whereas the moon, Mercury, and probably most of the satellites, are regarded as triaxial ellipsoids. However, the moon is the only celestial body for which it is possible to determine the parameters of rotation with admissible accuracy directly from observations.

In 1693 Cassini empirically discovered three laws which describe mean lunar rotation well.

1. The period of lunar rotation and its period of revolution about the earth are equal.
2. The inclination of the lunar equator to the ecliptic is constant.
3. The poles of the lunar equator, ecliptic and lunar orbit lie on a single great circle.

The first law indicates that there is a force which equalizes the periods of lunar rotation about its axis and revolution about the earth. The semimajor lunar axis is usually directed toward the earth. If lunar motion about the earth occurred in a circular orbit coinciding with the lunar equator, the maximum radius would always be directed toward the center of the earth. In actuality, the lunar orbit is noncircular, and therefore this maximum radius is not always directed toward the center of the earth, but is deflected from it.

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These laws make it possible to establish the so-called "prime meridian" from which longitude is reckoned on the moon. When the mean lunar longitude is equal to the mean longitude of the ascending node of its orbit, the plane of the lunar prime meridian passes through the center of the earth. The

intersection of the plane of the prime meridian with the lunar equatorial plane determines the "prime radius."

It is assumed that the moon, while still in a plastic state, was already a triaxial ellipsoid and that its periods of rotation and revolution were brought into equalization by tidal friction prior to its hardening. Numerous investigations have shown that the moon is distended in the direction of the earth in the form of a "steady tidal wave." Data obtained by Hoppmann give the following deviations of the lunar triaxial ellipsoid from a sphere: semimajor axis, + 8.8 km, middle semiaxis, situated approximately in the equatorial plane, + 1.1 km, and semiminor axis, almost coinciding with the lunar axis of rotation, - 4.7 km.

In addition, modern investigations by Soviet astronomers A. A. Yakovkin, and others, confirm the asymmetry in the lunar figure relative to the equatorial plane. Since the prime radius periodically is deflected by gravitational force from the direction to the center of the earth, the lunar body exhibits pendulum-like oscillations about its mean position. These oscillations are reflected in inequalities in lunar motion. Modern precise observations have revealed small deviations from Cassini's laws and these have been named lunar physical librations. These deviations are dependent to a lesser degree on the amplitude of inequalities in orbital motion, and to a greater degree on their period.

Approximately 100 years after the discovery of Cassini's laws, Lagrange and Laplace formulated a theory of lunar rotational motion. However, neither Laplace, nor Wichmann and Franz, who came later, derived convenient and practical formulas for processing observations.

Hayn [67-73] investigated this problem in adequate detail. He supplemented and somewhat refined the theory formulated by his predecessors. In his exposition this theory was more suitable for both familiarization purposes and for practical application. Accordingly, here we will give a concise outline of the theory of lunar physical libration as given by Hayn.

In order to determine the influence of terrestrial attraction on the rotational motion of the moon about its center of gravity it is necessary to

determine the position of the earth at a particular moment in time in the coordinate system of the principal lunar axes of inertia. We will place the origin of the moving rectangular coordinate axes at the center of lunar mass. The positive direction of the x -axis is along the prime radius; the positive direction for the y -axis is in the lunar equatorial plane in the direction travelled by the moon in order to form a right angle with the x -axis, and the positive direction of the z -axis is toward the north pole from the lunar equator. We will call this a selenographic coordinate system.

The selenographic coordinate system (x, y, z) relative to the fixed coordinate system (X, Y, Z) , connected to the plane of the ecliptic, is stipulated by the three Euler angles (Figure 2):

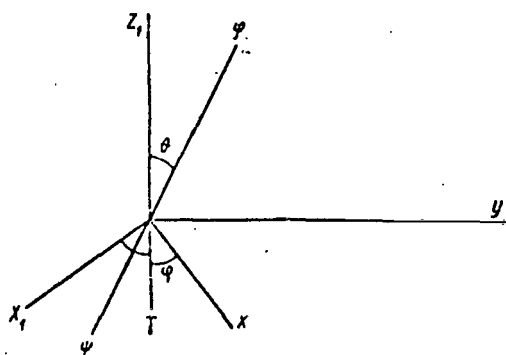


Figure 2

θ is the inclination of the lunar equator to the ecliptic, ϕ is the angle between the descending node of the lunar equator and the positive direction of the x -axis, ψ is the longitude of the lunar equator descending node. These parameters are unknown functions of time. We will introduce the following notations: M is the earth's mass, M_1 is the lunar mass, dm is an element of lunar mass, x, y, z

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are the coordinates of the earth's center of gravity, x', y', z' are the coordinates of an element of lunar mass (dm), R is the earth-moon distance, m is the mean lunar longitude, n is the longitude of the ascending node for the lunar orbit on the ecliptic, i is inclination of the lunar orbit to the ecliptic, I is the mean inclination of the lunar equator to the ecliptic. With these notations Cassini's laws are expressed by the following formulas:

$$\theta = I, \phi = 180^\circ, m - \psi = 180^\circ + m - n, \psi = n.$$

Under the influence of variable terrestrial gravitation, the inclination of the lunar orbit and the lunar velocity of rotation do not remain constant; in addition, the lunar equator descending node does not coincide with the ascending node of its orbit. Accordingly, the true value of the Euler angles θ , ϕ and ψ can be determined using the expressions

$$\theta = I + \rho, \quad \varphi = 180^\circ(m + \tau) - \psi, \quad \psi = n + \sigma, \quad (1)$$

where τ is physical libration in longitude, ρ is physical libration in inclination, σ is physical libration in node. These small parameters are functions of time and are obtained with integration of the Euler differential equations determining rotation of a solid body.

$$\begin{aligned} p &= -\sin \varphi \sin \theta \frac{d\psi}{dt} - \cos \varphi \frac{d\theta}{dt}, \\ q &= -\cos \varphi \sin \theta \frac{d\psi}{dt} + \sin \varphi \frac{d\theta}{dt}, \\ r &= \frac{d\varphi}{dt} \cos \theta \frac{d\psi}{dt}. \end{aligned} \quad (2)$$

$$\begin{aligned} A \frac{dp}{dt} + (C-B)qr &= z \frac{\partial N}{\partial y} - y \frac{\partial V}{\partial z}, \\ B \frac{dq}{dt} + (A-C)pr &= x \frac{\partial V}{\partial z} - z \frac{\partial V}{\partial x}, \\ C \frac{dr}{dt} + (B-A)pq &= y \frac{\partial V}{\partial x} - x \frac{\partial V}{\partial y}. \end{aligned} \quad (3)$$

Where A , B , C are the main lunar moments of inertia ($A < B < C$) relative to the coordinate axes x , y , z respectively (the right-hand sides of these equations contain the moments of external forces obtained with expansion of terrestrial gravitation potential); p , q , r are the components of the instantaneous angular velocity of lunar rotation relative to a coordinate system connected to the main axes of inertia or a system of selenographic coordinates close to it; V is the earth's potential.

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$$A = \int [(y')^2 + (z')^2] dm; \quad B = \int [(z')^2 + (x')^2] dm; \quad C = \int [(x')^2 + (y')^2] dm,$$

$$V = M \int [(x-x')^2 + (y-y')^2 + (z-z')^2]^{-\frac{1}{2}} dm.$$

In determining V it is possible to limit oneself to the first three terms of the expansion because the accuracy in determining the positions of the principal axes of inertia is up to $2''$, i.e., for the observer on earth, up to $0''.01$. Therefore

$$V = \frac{MM_1}{R} + \frac{1}{2} \frac{M}{R^3} (A+B+C) - \frac{3}{2} \frac{M}{R^5} (Ax^2 + By^2 + Cz^2).$$

Substituting this V value into formula (3), and introducing the notations

$$\frac{C-B}{A} = \alpha, \quad \frac{C-A}{B} = \beta, \quad \frac{B-A}{C} = \gamma, \quad (4)$$

we obtain

$$\begin{aligned} \frac{dp}{dt} + \alpha q r &= 3M\alpha R^{-5} yz, \\ \frac{dq}{dt} + \beta p r &= 3M\beta R^{-5} zx, \\ \frac{dr}{dt} + \gamma p q &= 3M\gamma R^{-5} xy. \end{aligned} \quad (5)$$

We note that α , β and γ are small parameters; their determination from observations is the fundamental problem. They are related by the expression

$$\alpha - \beta + \gamma - \gamma\beta\gamma \cong \alpha - \beta + \gamma = 0.$$

In equations (5) x , y and z must be expressed as functions of time. This requires that they be represented in an ecliptic coordinate system (ξ, η, ζ) , where ξ is directed to the lunar equator ascending node, and ζ is directed to the north pole; then we obtain

$$\begin{aligned} x &= \xi \cos \varphi + \eta \cos \theta \sin \varphi - \zeta \sin \theta \sin \varphi, \\ y &= -\xi \sin \varphi + \eta \cos \theta \cos \varphi - \zeta \sin \theta \cos \varphi, \\ z &= \zeta \cos \varphi + \eta \sin \theta. \end{aligned} \quad (6)$$

The position of the moon in the ecliptic system is given by the true longitude L and the local latitude B . Then we can determine the true selenocentric

longitude of the earth (v), visible from the moon and reckoned from the lunar equator descending node, i.e.

$$v = L + 180^\circ - \psi.$$

Since the earth's true latitude is $\sim B$, then

$$\xi = R \cos B \cos v, \eta = R \cos B \sin v, \zeta = -R \cos B \sin \theta.$$

Substituting these values into equations (6), we obtain formulas for transforming from (x, y, z) to (R, h, B) :

$$\begin{aligned} x &= R \cos B [\cos v \cos \varphi + \sin v \sin \varphi \cos \theta + \sin \theta \sin \varphi \sin \theta], \\ y &= R \cos B [-\cos v \sin \varphi + \sin v \cos \varphi \cos \theta + \sin \theta \cos \varphi \sin \theta], \\ z &= R \cos B [-\sin \theta \cos \theta + \sin v \sin \theta]. \end{aligned} \quad (7)$$

We express the variables as functions of time. The earth's mass is

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$$M = 4\pi^2 a^3 : T^2 (1 + M_1),$$

where M_1 is the lunar mass, expressed in fractions of the earth's mass (1:81.5) a is the semimajor axis of the lunar orbit, T is the lunar period of rotation in days. Since $2\pi:T$ is the mean diurnal motion (m') of the moon, when $a = 1$ we will have

$$M = 0.9878 (m')^2 \quad (8)$$

Using the lunar orbital motion it is possible to obtain the dependence for the parameters R, B, v, ϕ on time, and therefore the coordinates x, y and z can be represented as functions of time. The parameter I is assigned an approximate value; as a result, α, β and γ are also approximate. These values are then made more precise through observations. If the right-hand sides of equation (5) are expressed as functions of time, for the time being the unknown parameters τ, ρ and σ appear in the expansion. However, since they are very small, initially they can be neglected. Hansen gives the true lunar longitude (L), parallax, and sine of latitude B as functions of time using five arguments: g , the mean lunar anomaly; g' , the mean solar anomaly; ω , the

distance of the lunar perigee from the lunar orbit ascending node; ω' , the distance of the solar perigee from the lunar orbit ascending node; n , the longitude of the lunar orbit ascending node. These parameters can be determined using Hansen's data. A detailed description of how the Hansen expansions for true longitude, parallax and sine of latitude can be used in obtaining the functions R , $\cos B$, $\tan B$, v and the derivatives of yz , zx , xy is given in a study by Hayn [68].

In the computations it is necessary to take into account all the terms which affect the results. With multiplication of the series having sines and cosines, we obtain a new series with arguments of a different type. The determination of the period of pendulum-like lunar oscillations is of considerable importance. Accordingly, in the series for L , $1:R$ and $\sin B$ it is also necessary to retain those terms whose coefficients are small and which may seem unimportant.

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Hayn gives linear combinations of the arguments g , g' , ω , ω' and the values $\sin \text{Arg}$, $\cos \text{Arg}$ (Table 6).

TABLE 6

Argument	$L + g + \omega + n$	$1:R = 1,000000$
g	$+ 22641 \sin \text{Arg}$	$+ 0.05452 \cos \text{Arg}$
$2g$	$+ 769$	$+ 298$
$- g'$	$+ 668$	$- 12$
etc.		

and also the series $\sin B$. Denoting through $S = R^{-5}yz$, $T = -R^{-5}xz$, for which linear combinations of the arguments g , g' , ω and ω' were given, Hayn obtained the values given in Table 7.

TABLE 7

Argument	$- S$	T
$- g + \omega$	$- 0.00039 \cos \text{Arg}$	$+ 0.00026 \sin \text{Arg}$
ω	$- 640$	$+ 318$
$g + \omega$	56	$+ 11,561$
$2g - \omega$	$+ 628$	$+ 1,576$
etc.		

B - S also includes the group of terms

$$0,116 \tau \sin(g+\omega) + 0,109 \sin g [g \sin(g+\omega) - I \delta \cos(g+\omega)],$$

in T

$$\begin{aligned} &+ 0,9933 g \sin(g+\omega) - 0,9943 I \delta \cos(g+\omega) + \\ &+ 0,109 \sin g [g \cos(g+\omega) + I \delta \sin(g+\omega)] + \\ &+ 0,163 \cos g [g \sin(g+\omega) - I \delta \cos(g+\omega)]. \end{aligned}$$

R^{-5}_{xy} gives

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$$\begin{aligned} \sum H \sinh = &+ 0,10829 \sin g + \\ &+ 321 - g' - \\ &- 11 - g' + \omega - \omega' + \\ &+ 118 - 2g' + 2\omega - 2\omega' + \\ &+ 2125 - g - 2g' + 2\omega - 2\omega' - \\ &- 1 - 2\omega + \\ &+ 28 - 2g' - 2\omega' + \\ &+ 4 - n - 5^0. \end{aligned} \quad (9)$$

and

$$- 0,9852 \tau - 0,162 \tau \cos g - 0,045 I \delta [1 - \cos(2g + 2\omega)].$$

We give these same values as computed by Koziel [79]

$$\begin{aligned} \sum H \sinh = &+ 0,1082428 \sin g + \\ &+ 124,77 - 2g + \\ &+ 3193 - g' - \\ &- 11 - 2\omega - \\ &- 93 - g' + \omega - \omega' + \\ &+ 328 - 2g - 2\omega' + \\ &+ 17 - g' + 2\omega - 2\omega' + \\ &+ 1200 - 2g' + 2\omega - 2\omega' + \\ &+ 21546 - g - 2g' + 2\omega - 2\omega' + \\ &+ 14672 - 2g - 2g' + 2\omega - 2\omega' + \\ &+ 324 - 2g' - 2\omega'. \end{aligned}$$

It therefore can be seen that the H_i values obtained by Koziel differ from Hayn's by a value in the fifth decimal place. From equation (2)

$$r = \frac{d\varphi}{dt} + \cos\theta \frac{d\psi}{dt} = \frac{dm}{dt} + \frac{d\tau}{dt} - \frac{d\psi}{dt} + \cos\theta \frac{d\psi}{dt} = \frac{dm}{dt} + \frac{d\tau}{dt} - 2\sin^2 \frac{1}{2}\theta \frac{d\psi}{dt}.$$

Since I is a small angle, we can write

$$\frac{dr}{dt} = \frac{d^2\tau}{dt^2} - \frac{I}{2} \frac{d^2 I \delta}{dt^2}.$$

If we substitute $M' = 3M$, from the third equation in system (5) we have

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$$\begin{aligned} \frac{d^2\tau}{dt^2} + 0,9052 M' \gamma \tau = M' \gamma \sum H_i \sin h_i - \gamma p q + \\ + M' \gamma [-0,162 \tau \cos g - 0,045 I \delta (1 - \cos(2g + 2\omega))] + \frac{I}{2} \frac{d^2 I \delta}{dt^2}. \end{aligned} \quad (10)$$

This equation is not integrated in this complete form. Neglecting all terms other than the first on the right-hand side, we obtain a linear equation whose total integral is

$$\tau = A \sin(a + t \sqrt{0,9852 M' \gamma}) - \sum \frac{M' \gamma H_i \sin h_i}{h_i^2 - 0,9852 M' \gamma}. \quad (11)$$

Here h' is the mean diurnal change in the argument h , whose general form is $h = k_1 g + k_2 g' + k_3 \omega + k_4 \omega'$, where k are small numbers, t is time in mean days; the first term containing the arbitrary constants A and a expresses free libration, whereas the second expresses forced libration; H_i are definite numbers dependent on the known parameters (elements of the lunar orbit, etc.).

Now we will examine the terms neglected in (10), where $\gamma p q$ can be completely discarded because very obviously it is very small. In addition, it can be found that

$$\begin{aligned} - 0,162 \tau \cos g = + 0,00001 \sin 2g + 0,00003 \sin (-g - g') + \\ + 0,00003 \sin (g - g'), \end{aligned}$$

and

$$- 0,045 I \delta [1 - \cos(2g + 2\omega)] = + 0,00002 \sin g.$$

The last term can be written in the form

$$M'_{\gamma} \frac{I}{2M_{\gamma}} \frac{d^2 I \delta}{dt^2} = M'_{\gamma} [+ 0,01356 \sin g - 0,00454 \sin (g+2\omega) + \\ + 0,00555 \sin (2g+2\omega)] .$$

We also add to the series $\Sigma H \sin h$

$$+ 0.00002 \sin g u + 0.01356 \sin g$$

The diurnal change in the four arguments in fractions of radius will be as follows: + 0.2280273, + 0.0172022, + 0.0028686, + 0.0009250. Assuming $\gamma = 0.0001703$, $m' = 0.229972$, $M = 0.052242$, we obtain $0.9852 M'_{\gamma} = 0.00002629$. /77

For these $c(h')^2$ values the denominator in the second term (11) is equal to zero, i.e., for the argument whose diurnal motion is equal to 0.005127. The parameter 2ω comes closest to this value because its diurnal motion is + 0.005737. Accordingly, the argument 2ω is of great importance for lunar axial rotation. With the cited values, for τ we obtain the expression

$$\begin{aligned} \tau = & A \sin(a + 1057'' zt) - 13'' \sin g + \\ & + 65'' \sin g' + \\ & + 3'' \sin (-g' - \omega - \omega') - \\ & - 7'' \sin (-2g' + 2\omega - 2\omega') - \\ & - 3'' \sin (g - 2g + 2\omega - 2\omega') + \\ & + 8'' \sin 2\omega - \\ & - 1'' \sin (2g' - 2\omega') + \\ & + 8'' \sin (n - 5^\circ) . \end{aligned} \quad (12)$$

The integration of the two other equations in system (5) is considerably more complex. In place of the sought-for ρ and σ we introduce the variables x and y

$$x = \sin \varphi \sin \theta , \quad y = \cos \varphi \sin \theta . \quad (12')$$

Since $\phi = 180^\circ + g + \omega + \tau - \sigma$, and $I = 0.026859$, we can write

$$\begin{aligned} x = & -I \sin (g + \omega) - g \sin (g + \omega) - I \tau \cos (g + \omega) + I \delta \cos (g + \omega) , \\ y = & -I \cos (g + \omega) - g \cos (g + \omega) + I \tau \sin (g + \omega) - I \delta \sin (g + \omega) . \end{aligned}$$

These transformations can be made on the following basis. Since

$$\begin{aligned}p &= -\sin \varphi \sin \theta \frac{d\psi}{dt} - \cos \varphi \frac{d\theta}{dt}, \\q &= -\cos \varphi \sin \theta \frac{d\psi}{dt} + \sin \varphi \frac{d\theta}{dt}, \\ \psi &= 180^\circ + m \cdot \tau - \varphi, \\ \frac{d\psi}{dt} &= \frac{dm}{dt} + \frac{d\tau}{dt} - \frac{d\varphi}{dt}, \\ \frac{dm}{dt} &= m',\end{aligned}$$

Then

$$\begin{aligned}p &= m' \sin \varphi \sin \theta - \sin \varphi \sin \theta \frac{d\tau}{dt} + \sin \varphi \sin \theta \frac{d\varphi}{dt} - \cos \varphi \frac{d\theta}{dt}, \\q &= -m' \cos \varphi \sin \theta - \cos \varphi \sin \theta \frac{d\tau}{dt} + \cos \varphi \sin \theta \frac{d\varphi}{dt} + \sin \varphi \frac{d\theta}{dt}.\end{aligned}$$

Taking into account the notations (12'), we obtain

$$\begin{aligned}p &= m'x - x \frac{d\tau}{dt} - \frac{dy}{dt} - 2 \cos \varphi \sin^2 \frac{1}{2} \theta \frac{d\varphi}{dt}, \\q &= -m'y - y \frac{d\tau}{dt} + \frac{dx}{dt} + 2 \sin \varphi \sin^2 \frac{1}{2} \theta \frac{d\varphi}{dt}.\end{aligned}$$

Hence we can obtain dp/dt and dq/dt ; taking into account the value $x = m' + d\tau/dt - 2 \sin^2 1/2 \theta d\psi/dt$, we obtain without higher-order terms

$$\begin{aligned}\frac{dp}{dt} + \alpha q r &= -\frac{d^2 y}{dt^2} - \frac{dx}{dt} m' (1 - \alpha) - \alpha (m')^2 y - \frac{dx}{dt} \frac{d\tau}{dt} - x \frac{d^2 \tau}{dt^2} + \frac{d}{dt} \left(\frac{1}{2} l^2 x \right. \\ &\quad \left. \times \cos(g + \omega) \frac{d\varphi}{dt} \right), \\ \frac{dq}{dt} - \beta z g &= \frac{d^2 x}{dt^2} - \frac{dy}{dt} m' (1 - \beta) + \beta (m')^2 x - \frac{dy}{dt} \frac{d\tau}{dt} - y \frac{d^2 \tau}{dt^2} - \frac{d}{dt} \left(\frac{1}{2} l^2 \sin(g + \omega) \frac{d\varphi}{dt} \right),\end{aligned} \quad (13)$$

In these equations the three last terms can be disregarded because they represent small values. Then the first equation of system (5) can be integrated. In the expression for T (Table 7), another transformation must also be made. Since

$$x = -0.02686 \sin(g + \omega) - 0.0269 \tau \cos(g + \omega) - g \sin(g + \omega) + I \cos(g + \omega),$$

for the term T

$$+ 0.1156 i \sin(g + \omega) + 0.9933 g \sin(g + \omega) - 0.9943 \cos(g + \omega)$$

we can write:

$$+ 0.08892 \sin (g+\omega) + 0.9938x - 0.0267\tau \cos (g+\omega).$$

Then both equations of system (5) assume the form

$$\begin{aligned} \frac{d^2y}{dt^2} - \frac{dx}{dt} m'(1-\alpha) + \alpha(m')^2 y &= M'\alpha S + U, \\ \frac{d^2x}{dt^2} - \frac{dy}{dt} m'(1-\beta) + x(\beta(m')^2 + 0.9938\beta M') &= M'\beta(T + 0.9938x) + W, \end{aligned} \quad (14)$$

where

$$\begin{aligned} U &= -\frac{dx}{dt} \frac{d\tau}{dt} - x \frac{d^2\tau}{dt^2} + \frac{d}{dt} \left(\frac{1}{2} I^2 \cos(g+\omega) \frac{d\varphi}{dt} \right), \\ W &= \frac{dy}{dt} \frac{d\tau}{dt} + y \frac{d^2\tau}{dt^2} + \frac{d}{dt} \left(\frac{1}{2} I^2 \sin(g+\omega) \frac{d\varphi}{dt} \right). \end{aligned}$$

The terms in S and $T + 0.9938x$ containing ρ , σ and τ can be disregarded in the first approximation; in the second approximation we have

$$\begin{aligned} -0.116\tau \sin (g+\omega) &= -0.00004 \cos (-g+\omega) + \\ + 0.109 \sin g [g \cos (g+\omega) - I \cos (g+\omega)] &+ 0.00001 \cos \omega + \\ &+ 4 \cos (g+\omega) - \\ &- 1 \cos (2g-\omega) - \\ &- 2 \cos (g-g'+\omega) + \\ &+ 2 \cos (g+g'+\omega), \\ + 0.109 \sin g [g \cos (g+\omega) + I \sin (g+\omega)] &= \\ = -0.00004 \sin (-g+\omega) + 0.163 \cos g [g \sin (g+\omega) - I \cos (g+\omega)] + \\ + 0.00001 \sin \omega - 0.0267\tau \cos (g+\omega) - 0.00008 \sin (g+\omega). \end{aligned}$$

Assuming $M'\alpha = 0.0000723$, $M'\beta = 0.0000988$, we obtain

$$\begin{aligned} \frac{U}{M'\alpha} &+ 0.00001 \cos (g+\omega) - \\ &- 87 \cos (2g+\omega) + \\ &+ 4 \cos (3g+3\omega) + \\ &+ 21 \cos (g-g'+\omega) + \\ &+ 21 \cos (g+g'+\omega), \end{aligned}$$

$$\begin{aligned} \frac{W}{W\beta} = & -0.00001 \sin(q+\omega) - \\ & - 64 \sin(2q+\omega) + \\ & + 3 \sin(3q+3\omega) + \\ & + 15 \sin(q-q'+\omega) + \\ & + 15 \sin(q+q'+\omega). \end{aligned}$$

These terms are added to S and $T + 0.9938x$. The right-hand sides of equations (14) are series whose terms have the form $F \cos g$ and $G \sin g$. Proceeding to integration, we replace the right-hand zero. We obtain a solution for the systems: ✓80

$$\begin{aligned} \text{i)} \quad x &= A_1 \cos \lambda t, \quad y = B_1 \sin \lambda t, \\ \text{ii)} \quad x &= A_2 \sin \lambda t, \quad y = B_2 \cos \lambda t. \end{aligned}$$

Substituting them into (14), we obtain a fourth degree equation for λ ; its two real roots are equal to

$$\begin{aligned} \lambda_1 &= m' \left[1 + \frac{\varepsilon - 1}{2} \beta \right], \\ \lambda_2 &= m' \sqrt{\varepsilon \alpha \beta}, \end{aligned} \tag{15a}$$

where $\varepsilon - 1$ is the product of the three cofactors 3, 0.9878, and 0.9938, i.e., 2.9450. Introducing the four integration constants related by a definite dependence with A_1, A_2, B_1, B_2 , we find the integral for the system of equations (14):

$$\begin{aligned} x &= B \sin(\theta + \lambda_1 t) + C \sin(C + \lambda_2 t), \\ y &= B \cos(\theta + \lambda_1 t) - \sqrt{\frac{EA}{\alpha}} C \cos(C + \lambda_2 t). \end{aligned} \tag{15b}$$

Now in order to obtain the total integral of the equation, we take into account that the equation is satisfied by the values

$$x = X \sin g, \quad y = Y \cos g.$$

Hence we obtain

$$\begin{aligned} x' &= Xg' \cos g, & y' &= -Yg' \sin g, \\ x'' &= -X(g')^2 \sin g, & y'' &= -Y(g')^2 \cos g. \end{aligned}$$

Then we obtain the characteristic equation

$$\begin{aligned} -X(g')^2 + Yg'm'(1-\beta) + X\beta(m')^2 \varepsilon &= G, \\ -Y(g')^2 + Xg'm'(1-\alpha) + Y\alpha(m')^2 &= F \end{aligned}$$

or

$$\begin{aligned} -X[(g')^2 - \beta(m')^2 \varepsilon] + Yg'm'(1-\beta) &= G, \\ -Y[(g')^2 - \alpha(m')^2] + Xg'm'(1-\alpha) &= F, \end{aligned}$$

and hence we obtain

$$\begin{aligned} X &= \frac{-G[(g')^2 - \alpha(m')^2] - Fg'm'(1-\beta)}{[(g')^2 - \beta(m')^2 \varepsilon][(g')^2 - \alpha(m')^2] - (g')^2(m')^2(1-\alpha)(1-\beta)}, \\ Y &= \frac{-F(g' - \beta m'^2 \varepsilon) - Gg'm'(1-\alpha)}{[(g')^2 - \beta m'^2 \varepsilon](g')^2 - \alpha(m')^2 - (g')^2(m')^2(1-\alpha)(1-\beta)}, \end{aligned} \quad (16)$$

where g' is the diurnal change in the considered argument. Using formulas (15) and (16) we can compute the general integrals for both differential equations, provided that α and β are known. The four integration constants must be determined from observations. By definition, $x = \sin \phi \sin \theta$, $y = -\cos \phi \sin \theta$, it follows that

$$\begin{aligned} x &= \sin(q + \omega + \tau - \sigma) \sin(I + \rho), \\ y &= -\cos(q + \omega + \tau - \sigma) \sin(I + \rho). \end{aligned}$$

Assuming $\cos(\tau - \sigma) = 1$, $\sin(I + \rho) = I + \rho$, and disregarding the product $\rho(\tau - \sigma)$, we obtain

$$\begin{aligned} x &= -I \sin(q + \omega) - \rho \sin(q + \omega) - I \tau \cos(q + \omega) - I \sigma \cos(q + \omega), \\ y &= I \cos(q + \omega) - \rho \cos(q + \omega) + I \tau \sin(q + \omega) - I \sigma \sin(q + \omega). \end{aligned}$$

hence

$$\begin{aligned} I + g &= -x \sin(g + \omega) - y \cos(g + \omega), \\ I\sigma - Ix &= x \cos(g + \omega) - y \sin(g + \omega). \end{aligned} \quad (17)$$

The integrals (14) now contain for the argument $g + \omega$ the values X and Y , which we will denote as $X_{g+\omega}$, $Y_{g+\omega}$. Hence I and σ are known, ρ and $I\sigma$ can be computed. Since x is expanded in a series with sine terms and y in a series with cosine terms, in ρ all the coefficients with $\cos$, and in $I\sigma$ with $\sin$ will be multiplied. From (17) we have

$$g = -I - x \sin(g + \omega) - y \cos(g + \omega),$$

where the argument $g + \omega$ gives the following terms

$$-X_{g+\omega} \sin^2(g + \omega) - Y_{g+\omega} \cos^2(g + \omega)$$

or

$$-X_{g+\omega} \frac{1 - \cos(2g + 2\omega)}{2} - Y_{g+\omega} \frac{1 + \cos(2g + 2\omega)}{2}.$$

This means that if we disregard other terms with small coefficients, insignificant for this particular case, then we have

$$2g = -2I - X_{g+\omega} - Y_{g+\omega} + X_{g+\omega} \cos(2g + 2\omega) - Y_{g+\omega} \cos(2g + 2\omega).$$

Since ρ consists only of periodic terms, then

$$X_{g+\omega} + Y_{g+\omega} = -2I. \quad (18)$$

This equation gives the dependence between I and the unknown parameters α and β . Assuming

$$\begin{aligned} I &= 0.026859, & t_{g+\omega} &= +0.00061 M' \alpha, \\ G_{g+\omega} &= +0.03884 M' \beta, & M' &= 0.156725, \\ \epsilon &= 3.9450 \end{aligned}$$

and denoting $\alpha:\beta = f$, we obtain

$$\beta = + 0.0006328 - 0.00528 f \beta + 1.0074 \beta^2$$

If on the right-hand side we substitute $\beta = 0.000631$, then

$$\beta = 0.0006328 - 0.00000293 f. \quad (19)$$

If I and f were determined from observations, the problem would be solved. On the basis of Hayn's determinations, we find that $I = 1^\circ 32' 20''$ and $f = 0.73$; it therefore follows that $\beta = + 0.0006307$, $\alpha = + 0.0004604$, $\gamma = + 0.0001703$. Using formulas (15), (16) and (17), we can compute the following values for the coefficients ρ and $I\sigma$:

$$\begin{aligned} \rho = & -8 \cos (\theta - 146'' 6 t) + & I\sigma = & I\tau + 8 \sin (\theta - 146'' 6 t) - \\ & + 0.662 C \cos (c + 50'' 8 t - g + \omega) + & - 0.662 C \sin (c + 50'' 8 t - g + \omega) + \\ & + 0.662 C \cos (c + 50'' 8 t + g + \omega) - & + 1.662 C \sin (c + 50'' 8 t + g + \omega) - \\ & - 106'' \cos g + & - 108'' \sin g + \\ & + 35'' \cos (g + 2\omega) - & + 35'' \sin (g + 2\omega) - \\ & - 11'' \cos (2g + 2\omega) - & - 11'' \sin (2g + 2\omega) - \\ & - 3'' \cos (2g' + 2\omega') - & - 3'' \sin (2g' + 2\omega') - \\ & - 2'' \cos (g - 2g' + 2\omega - 2\omega') - & - 2'' \sin (g - 2g' + 2\omega - 2\omega'). \end{aligned} \quad (20)$$

Thus, we have obtained τ , ρ and $I\sigma$. The terms in τ with the integration constant A and α are obviously small. The integration constants B and C have not yet been determined. These integration constants are variations called "free libration", in contrast to the other terms whose coefficients are determined by orbital motion and the configuration of the ellipsoid of inertia and therefore called "forced libration."

Free libration can be explained on a physical basis in the following way. If it is assumed that its coefficients are equal to zero, the motion of the ellipsoid is represented by formulas (1) and "forced libration" is determined by formulas (12) and (20). If some force displaces the ellipsoid from equilibrium, it will exhibit pendulum-like oscillations which can only be described by the "free libration" terms. One of the causes of such

oscillations may be a nonuniform distribution of mass in the moon or the impacts of colliding meteors with a large mass. Assuming that such oscillations formerly appeared, they obviously should also exist at the present time. Hayn gives the values of the coefficients for the functions τ , ρ and $I\sigma$ for different f values, as follows:

f	0.5	0.6	0.7	0.8
τ	-22"	-18"	-14"	-10" $\sin g$
	+132	+102	+74	+47 $\sin g'$
	-6	-5	+3	+2 $\sin(g' + \omega - \omega')$
	-14	-11	-8	-5 $\sin(-2g' + 2\omega - 2\omega')$

(21)

f	0.5	0.6	0.7	0.8
	-6	-4	-3	-2 $\sin(g - 2g' + 2\omega - 2\omega')$
	-6	-13	+16	+3 $\sin 2\omega$
g	+8	+8	+8	+8 $\sin(n - 5^\circ)$
g	-88"	-90"	-103"	-111" $\cos g$
	+9	+20	+32	+43 $\cos(g + 2\omega)$
	-11	-11	-11	-11 $\cos(2g + 2\omega)$
	-3	-3	-3	-3 $\cos(2g' + 2\omega')$
	-2	-2	-2	-2 $\cos(g - 2g' + 2\omega - 2\omega')$

(22)

f	0.5	0.6	0.7	0.8
$I\sigma$	-90	-98	-105	-113 $\sin g'$
	+9	+20	+32	+43 $\sin(g + 2\omega)$
	-11	-11	-11	-11 $\sin(2g + 2\omega)$
	-3	-3	-3	-3 $\sin(2g' + 2\omega')$
	-2	-2	-2	-2 $\sin(g - 2g' + 2\omega - 2\omega')$
	+4	+3	+2	+1 $\sin g'$

(23)

These expansions were performed independently in 1917 by Jonsson [76]. The $I\sigma$ and ρ expansions obtained by Hayn and Jonsson coincide. With respect to τ , a discrepancy appeared here in the sign of the coefficient of the term with

$\sin 2\omega$. In 1949 Koziel made independent computations for checking this discrepancy and for the most part obtained the same results as Hayn: $I\sigma$ and ρ are similar but the τ values differ.

f	0.5	0.6	0.7	0.8
τ	-22"	-18"	-14"	-10" <i>sin g</i>
	+132	+101	+73	+47 <i>sin g'</i>
	- 8	-17	+19	+ 4 <i>sin 2\omega</i>
	-14	-11	- 8	- 5 <i>sin (-2g'+2\omega-2\omega')</i>
	+ 8	+ 8	+ 8	+ 8 <i>sin (n-5^\circ)</i>

The coefficient of $\sin 2\omega$ for $f = 0.65$ is equal to $- 59''$; for $f = 0.67$, it is equal to $+ 84''$; and for $f = 0.662$, it is equal to infinity. The investigations revealed that with an increase in f this coefficient increases in absolute value. When $f = 0.662$ it is equal to infinity. A discontinuity occurs at this point and thereafter the coefficient decreases to $4''$ when $f = 0.80$.

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The differential equation written for τ is complex. After expansion of the right-hand sides of equation (5) in series, Hayn and Koziel, dropping a large number of terms, reduce it to the form

$$\frac{d^2\tau}{dt^2} + k^2\tau = M'g \sum H_i \sin h_i,$$

where k is dependent on f . The arguments h_i contain terms which vary proportionally to time. The general solution of the equation has the form

$$\tau = A \sin(a \cdot kt) - M'g \sum \frac{H_i \sin h_i}{\left(\frac{dh_i}{dt}\right)^2 - k^2}.$$

This equation makes sense only when $dh_i/dt \neq k$; otherwise there is a resonance phenomenon, i.e., the argument 2ω has a rate of change close to the coefficient k and equal to it when $f = 0.662$.

Until now we have considered only the effect of terrestrial gravitation on lunar rotation. However, the solar effect is also important. Solar attraction exerts an effect on lunar orbital and rotational motion; this is indicated by the presence of g' and ω' , which pertain only to the sun, whereas

in τ , the term dependent on solar perturbation is the largest. The direct effect of solar attraction on lunar rotation is proportional to the solar mass and inversely proportional to the third power of distance; thus, the solar effect is approximately 183 times less than that of the earth. It therefore follows that the small additional terms entering into the expressions for τ , ρ , and $I\sigma$, with solar attraction taken into account, are not observed from the earth. Since solar attraction is small, it can be computed by the introduction of an expression containing the solar coordinates in the right-hand sides of equations (5) in place of the earth's mass.

Assume that the solar mass is M'' , the selenocentric longitude of the sun is L'' , the selenocentric latitude of the sun is B'' , and the moon-sun distance is R'' ; then $M'' = 32,800/391.5^3$ and using tables for the sun we obtain:

$$\begin{aligned} L'' &= g + \omega + \omega' + 0.0003 \sin 2g', \\ R'' &= 1.000 + 0.0167 \cos g' + 0.0003 \cos 2g' - 0.002 \cos (-g' + \omega - \omega') + \\ &\quad + 0.0026 \cos (g - g' + \omega + \omega'), \\ B'' &= -0.0002 \sin (g + \omega). \end{aligned}$$

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Only the most important terms are included in the right-hand sides of equations (10) and (14)

$$\begin{aligned} -3M''\gamma 0.4994 \sin (2g - 2g' + 2\omega - 2\omega') &= -M''\gamma 0.00271 \sin (2g - 2g' + 2\omega - 2\omega'), \\ -3M''\alpha [0.0133 \cos (g - 2g' + \omega - 2\omega') \cdot 0.5\gamma] &= -M''\alpha [0.0001 \cos (g - 2g' + 2\omega - 2\omega') + 0.0027\gamma] \\ -3M''\beta [0.0133 \sin (g - 2g' + \omega - 2\omega') \cdot 0.5x] &= -M''\beta [0.00001 \sin (g - 2g' + 2\omega - 2\omega') + 0.0027x]. \end{aligned}$$

Integration of these additional expressions gives libration oscillations of less than $1''$. The x and y can be replaced by $x = -0.02686 \sin (g + \omega)$ and $y = 0.02686 \cos (g + \omega)$; then we obtain the values

$$0.00007M''\alpha \cos (g + \omega) u + 0.00007M''\beta \sin (g + \omega).$$

In order to determine the solar effect these values must be added to the right-hand side of equation (14); as a result there is virtually no change in the values $F_{g+\omega}$, $G_{g+\omega}$, $X_{g+\omega}$, $Y_{g+\omega}$ and the conditional equations (18).

Examining terrestrial attraction separately from equation (18), we obtain the following dependence between I , f , β :

$$\begin{aligned} &7.170739\beta + 5.007460f\beta - 7.176300f\beta^2 + \\ &+ 7.719265\beta^2I - 7.914592f\beta^2 \cdot 2I - 2I \cdot 5.356179 = 0 \end{aligned} \quad (24)$$

with allowance for solar attraction, we will have

$$\begin{aligned} &7.171081\beta + 5.054639f\beta - 7.177465f\beta^2 + \\ &+ 7.919265 \cdot 2I - 7.914592f\beta^2 \cdot 2I - 2I \cdot 5.356179 = 0. \end{aligned} \quad (25)$$

In the first case, when $I = 0.026859$ and $f = 0.73$ for β we obtain 0.0006307; in the second case we obtain 0.0006300.

The β , τ , ρ and $I\sigma$ values computed with the latter β value will differ insignificantly from the earlier determined values, i.e., these computations need not be made. However, if the inclination of the lunar axis is to be determined under the influence of the earth alone, from equation (24) we obtain $I = 1^\circ 32' 13''$. It can therefore be seen that I , with allowance for solar attraction, is increased by $7''$.

On the basis of the described theory, using observations we can find the parameters of lunar physical libration. Equations (1), (12) and (20) determine the motion of the lunar ellipsoid of inertia about the center of gravity, whose orbital motion is known. If I , f , A , a , B , b , C , c , were precisely known, it would be possible to compute the position of the ellipsoid reliably for any moment in time. However, these constants are determined from observations, and therefore approximate values are known for I and f , whereas until now it has not been possible to determine A , B and C , because they are small.

CHAPTER IV

DETERMINING THE PARAMETERS OF LUNAR ROTATION BY A METHOD NOT DEPENDENT ON ITS PROFILE

The parameters of lunar rotation have been determined for more than a hundred years primarily by the Bessel method from heliometric measurements of distances and position angles of several points uniformly distributed along the illuminated lunar limb relative to the crater Moesting A. This method (except for the reduction method developed by Hayn and A. A. Yakovkin and the rigorous processing of observations by Koziel) has changed little up to the present time.

A shortcoming of the Bessel and Koziel method is the difficulty in taking into account the relief of the continuously changing complex lunar profile. /88
Accordingly, it is necessary to seek new observation methods and new methods for processing observations. Two approaches are possible. The first is finding the true figure of the moon with some idealization of its complex physical surface which should be stipulated by relief maps. On these maps the elevations in all sectors are reckoned from a spherical surface of selected radius; the center of the latter coincides with the lunar center of mass and the radius is a constant value. Such a map would solve the problem of processing heliometric and other observations. However, such a map does not yet exist. It is necessary to construct such a map, although the maps compiled by A. A. Nefed'yev are a considerable advance in studying the lunar profile; however, they do not reflect its asymmetry relative to the central meridian.

The second approach is finding an observation method in which the observer would not have to be concerned with the lunar limb, and therefore, in processing observations (measurements) it would not be necessary to have a knowledge of the physical relief of the apparent lunar limb. One of the possible methods meeting this requirement was proposed in 1961 by A. A. Yakovkin. The fundamental idea in this method is measuring the position angles of features situated within the visible lunar disk (in the limb zone) near the crater Moesting A, or measuring the distances from the crater to

features in the limb zone. In actuality, these angles and distances must be sensitive to changes in physical libration. They must exhibit periodic oscillations whose approximate amplitude is easy to compute prior to observations.

Libration in longitude is reflected most appreciably in observations of the position angles of near-polar craters, whereas libration in latitude is manifested most clearly when observing craters near the equator. However, it is also possible to measure the distances between the crater Moesting A and limb craters. In this case the lunar physical libration in longitude is manifested most strongly when a limb crater is situated near the lunar equator, whereas libration in latitude is manifested most clearly when observing craters selected near the lunar poles. Thus, it is necessary to select two craters (one near the pole and the other near the equator) in order to determine the parameters of lunar rotation with assurance, i.e., ascertain the fundamental sought-for parameters I and f .

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Observations are processed using formulas in which the following notations are employed: λ, β are the selenographic coordinates of the crater, l', b' are the selenocentric ecliptic coordinates of the crater, λ, β' is the observer's position (on earth) on the selenocentric sphere, the ecliptic coordinates, m is the mean lunar longitude, n is the mean longitude of the ascending node of the lunar orbit on the ecliptic, I_0 is the mean inclination of the lunar equator to the ecliptic (approximate value).

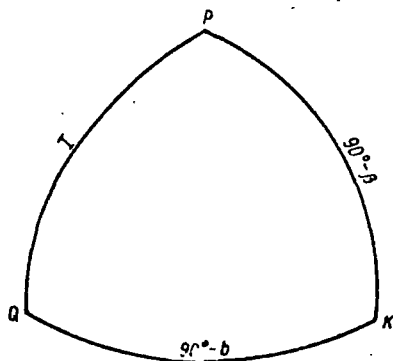


Figure 3

The indicated ecliptic longitudes are computed from the mean equinox of the particular time.

We will examine the triangle (Figure 3):

with the vertices: P is the pole of the lunar equator, Q is the pole of the ecliptic and K is the position of the crater. According

to Cassini's laws, without allowance for lunar physical libration, we obtain:

Angle	Side
$Q = l - n - 90^\circ$	$PQ = I$
$P = 90^\circ - (\lambda + m - n)$	$QK = 90^\circ - \delta$
	$PK = 90^\circ - \beta$

Hence, we obtain the following equations:

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$$\begin{aligned} \cos \delta \cos(l-n) &= -\cos \beta \cos(\lambda + m - n), \\ \cos \delta \sin(l-n) &= \sin \beta \sin I - \cos \beta \cos I \sin(\lambda + m - n), \\ \sin \delta &= \sin \beta \cos I + \cos \beta \sin I \sin(\lambda + m - n), \end{aligned} \quad (26)$$

from which the selenocentric ecliptic coordinates of the crater l and B are computed. Since I is assumed to be a constant value, whereas β is constant for the crater, from these equations it is possible to compile a table with the adopted I value and the argument $x = \lambda + m - n$. For example, for the crater Moesting A

$$\begin{aligned} \cos \delta \cos(l-n) &= -0.998445 \cos(\lambda + m - n), \\ \cos \delta \sin(l-n) &= -0.001508 - 0.998080 \sin(\lambda + m - n), \\ \sin \delta &= -0.055715 + 0.027007 \sin(\lambda + m - n). \end{aligned}$$

In order to determine l and B it is necessary to add the influence of physical libration on the coordinates. It was assumed that $f_0 = 0.85$, $I_0 = 1^\circ 33' 0''$. The well-known formulas

$$\begin{aligned} \Delta l &= (\infty) \tau_0 \sec \delta + \lg \delta \sin(l-n) \tau_0 \sin I + \lg \delta \cos(l-n) g_0, \\ \Delta \delta &= -\sin I \cos(l-n) (\tau_0 - \bar{\tau}_0) - \sin(l-n) g_0. \end{aligned}$$

are used for this purpose.

Thus, we obtain the selenocentric ecliptic coordinates of craters, corrected for the influence of physical libration: $l' = l + \Delta l$ and $B' = B + \Delta B$.

In order to compute the observer's position (earth) on the selenocentric sphere, obtaining the ecliptic coordinate λ' and β' , it is necessary to know

the apparent lunar coordinates α' , δ' . In this procedure the first step is to interpolate from the Astronomical Almanac the geocentric coordinates of the moon α , δ , its radius h_0 and parallax π at the observation times. Parallactic changes in coordinates and radius were computed using formulas which were reduced to a form convenient for computations on an electronic computer:

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$$\tan(\alpha' - \alpha) = - \frac{\rho \cos \varphi' \sec \delta \sin \pi \sin t}{1 - \rho \cos \varphi' \sec \delta \sin \pi \cos t}, \quad (27)$$

$$\tan(\delta' - \delta) = - \frac{\rho \cos \varphi' \sin \pi (\tan \phi' \cos \delta - \gamma \sin \delta)}{1 - \rho \cos \varphi' \sin \pi (\tan \phi' \sin \delta + \gamma \cos \delta)},$$

$$\frac{h' - h}{h} = \tan(\delta' - \delta) \left[\frac{\varphi' \sin \delta \cdot \gamma \cos \delta}{\varphi' \cos \delta - \gamma \sin \delta} + \frac{\tan(\delta' - \delta)}{2} \right],$$

Here

$$\gamma = \cos t + \frac{1}{2} \sin t \tan(\alpha' - \alpha),$$

where π is the lunar horizontal parallax, ρ and ϕ' are the radius vector and geocentric latitude of the place of observation respectively; t is the hour angle.

We will give the $\rho \cos \phi'$ and $\tan \phi'$ values for those places where either photographic or visual observations used in our study were made (Table 8).

TABLE 8

Place of Observation	$\rho \cos \phi'$	$\tan \phi'$
Main Astronomical Observatory Academy of Sciences Ukrainian SSR	0.6331784	1.199228
Kiev Astronomical Observatory	0.637996	1.202980
Kislovodsk Mountain Station	0.7235948	1.9507549

Using the determined values $\alpha' - \alpha$, $\delta' - \delta$, and $h' - h$, we computed the topocentric values of the coordinates α' , δ' and h' for the moon (see Appendix I).

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$$\cos \beta \sin \lambda = \sin \epsilon \sin \delta' + \cos \epsilon \cos \delta' \sin \alpha',$$

$$\cos \beta \cos \lambda = \cos \delta' \cos \alpha'.$$

$$\lambda' = \lambda + 180^\circ,$$

$$\beta' = -\beta. \quad (28)$$

We will examine on a sphere drawn about the center of the moon a triangle with the following vertices: Q , the pole of the ecliptic; K , the crater; T , the center of the lunar disk, i.e., the projection of the place of observation (Figure 4).

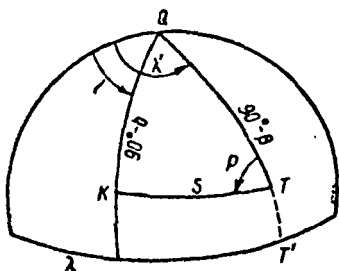


Figure 4

We denote TK by S , and the position angle by P . On a photographic plate the arc QT is represented by a straight line and the position angle P is reckoned from the north through the east. Using Figure 4 we find

$$\begin{aligned} \cos S &= \sin \beta' \sin \delta' + \cos \beta' \cos \delta' \cos (\lambda' - l') = x g^{-1}, \\ \sin S \cos P &= \cos \beta' \sin \delta' - \sin \beta' \cos \delta' \cos (\lambda' - l') = x g^{-1}, \\ \sin S \sin P &= \cos \delta' \sin (\lambda' - l') = y g^{-1}. \end{aligned} \quad (29)$$

Using as the origin of the rectangular coordinates the lunar center O , the z -axis is directed toward the observer at the point T ; the x -axis is oriented along the projection of the normal at Z to the plane of the ecliptic; the y -axis is directed to the east; $\rho = 0.272506$ (Figure 5).

The corrections for the assumed approximate values of inclination of the lunar equator to the ecliptic (I), the functions of the lunar moments of inertia (f), and the selenographic coordinates of the craters which were involved in the preceding computations will now be regarded as differentials.

The equation (31) must be differentiated in order to find the dependence between the differentials of the position angles and the mentioned differentials. A physical libration corresponding to an approximate value $f_0 = 0.85$ was added to the l and B values, computed using formula (26). In order to transform to the value $f = 0.85 + df$, we will assume

$$dm = \Delta\tau, dn = \Delta\delta, dI = \Delta I, \Delta g.$$

After differentiating equation (26), with these substitutions we obtain

$$\begin{aligned} dl &= \Delta\delta + (\alpha)(\Delta\tau - \Delta\sigma) \sec b + (\delta)(\Delta I + \Delta g) \sec b = A dI + B df + h d\alpha + K d\beta, \\ db &= -(\beta)(\Delta\tau - \Delta\sigma) - (\gamma)(\Delta I + \Delta g) = A' dI + B' df + L' d\alpha + K' d\beta. \end{aligned} \quad (32)$$

Here for brevity we use the notations

$$\begin{aligned} (\alpha) &= \cos I \cos b - \sin I \sin b \sin(l-n), \\ (\beta) &= \sin I \cos(l-n), \\ (\gamma) &= \sin(l-n), \\ (\delta) &= \sin b \cos(l-n). \end{aligned}$$

The values of physical libration in longitude τ , in inclination ρ , and in node σ are dependent on the value $f = (C - B)/(C - A) \cdot B/A$, which in turn is a sought-for unknown. In place of τ , ρ and σ , in the conditional equations we substitute the following expressions: /95

$$\tau_0 + \left(\frac{d\tau}{df}\right)_0 \Delta f, \rho_0 + \left(\frac{d\rho}{df}\right)_0 \Delta f, \sigma_0 + \left(\frac{d\sigma}{df}\right)_0 \Delta f.$$

In this case τ_0 , ρ_0 , σ_0 and their derivatives correspond to the assumed initial f_0 value for which the correction Δf is sought.

In this study the initial f_0 value was assumed equal to 0.85; therefore

$$\tau_0 = -6'' \sin g + 36'' \sin g' + 2'' \sin 2\omega - 3'' \sin(2g' + 2\omega - 2\omega') + 8'' \sin(n - 5^\circ),$$

$$\varrho_0 = -115'' \cos g + 49'' \cos(g + 2\omega) - 11'' \cos(g + 2\omega'),$$

$$I\tau_0 = -117'' \sin g + 49'' \sin(g + 2\omega) - 11'' \sin(g + 2\omega')$$

and their derivatives

$$\frac{d\tau}{dg} = 47'' \sin g - 240'' \sin g' - 16'' \sin 2\omega + 31'' \sin(-2g' + 2\omega - 2\omega'),$$

$$\frac{d\varrho}{dg} = -79'' \cos g + 115'' \cos(g + 2\omega),$$

$$I\left(\frac{d\varrho}{dg}\right) = -79'' \sin g + 115'' \sin(g + 2\omega),$$

where g is the mean lunar anomaly, g' is the mean solar anomaly, ω is the distance of the lunar perigee from the lunar orbit ascending node, ω' is the distance of the solar perigee from the lunar orbit ascending node (Appendix, Table 2).

Taking into account that the left sides of the two last equations in group (29) give values of the x and y coordinates divided by ρ , we will differentiate them, assuming t and b to be variables

$$\begin{aligned} dx &= -\sin \beta' \cos b' \sin(\lambda' - l') g dt + [\cos \beta' \cos b' + \sin \beta' \sin b' \cos(\lambda' - l')] g db, \\ dy &= -\sin b' \sin(\lambda' - l') g db - \cos b' \cos(\lambda' - l') g dt, \end{aligned} \quad (33)$$

where x and y are orthogonal projections. The differentials of the x' and y' coordinates in an oblique projection do not differ from the differentials of the orthogonal coordinates, since the differential of the factor $1 + \cos S \sin h$ can be disregarded due to the small $\sin h$ values.

It is necessary to determine the position angle differentials. In order to do this, assuming all the parameters to be variables, differentiating equation (31), we obtain

$$-S \sin p_{1,2} dp_{1,2} + \cos p_{1,2} dS = dx'_2 - dx'_1,$$

$$S \cos p_{1,2} dp_{1,2} + \sin p_{1,2} dS = dy'_2 - dy'_1,$$

where x_2, y_2 denote the computed coordinates for the limb craters, x_1, y_1 are the coordinates of the crater Moesting A. Since we make observations in an equatorial coordinate system, whereas computations are made in an ecliptic system, it is necessary to compute the angle $\rho_{1,2}$ between the circle of latitude and the circle of declination (Appendix, Table 3) for transformation from the equatorial coordinate system to the ecliptic system or vice versa. After excluding ds from these equations, we obtain

$$d\rho_{1,2} = E(dx'_2 - dx'_1) + F(dy'_2 - dy'_1), \quad (34)$$

where

$$E = -\frac{\sin \rho_{1,2}}{S}, \quad F = \frac{\cos \rho_{1,2}}{S}.$$

After excluding $d\rho$ from these same equations, we obtain

$$dS = \cos \rho_{1,2} (dx'_2 - dx'_1) + \sin \rho_{1,2} (dy'_2 - dy'_1), \quad (35)$$

or

$$dS = E'(dx'_2 - dx'_1) + F'(dy'_2 - dy'_1),$$

where

$$E' = \cos \rho_{1,2}, \quad F' = \sin \rho_{1,2}.$$

Substituting the expressions for these differentials into the others in reverse order, we can analytically obtain the expressions

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$$\frac{\partial \rho_{1,2}}{\partial I}, \frac{\partial \rho_{1,2}}{\partial f}, \frac{\partial \rho_{1,2}}{\partial \lambda}, \frac{\partial \rho_{1,2}}{\partial \beta}, \frac{\partial S_{1,2}}{\partial I}, \frac{\partial S_{1,2}}{\partial f}, \frac{\partial S_{1,2}}{\partial \lambda}, \frac{\partial S_{1,2}}{\partial \beta}.$$

After differentiating equations (26) and (29) we obtain:

$$dI = A dI_0 + B df_0 + L d\lambda + K d\beta,$$

$$d\delta = A' dI_0 + B' df_0 + L' d\lambda + K' d\beta,$$

$$dx' = C dI + D d\delta,$$

$$dy' = C' dI + D' d\delta,$$

$$d\rho_{1,2} = E(dx'_2 - dx'_1) + F(dy'_2 - dy'_1),$$

$$dS_{1,2} = E'(dx'_2 - dx'_1) + F'(dy'_2 - dy'_1).$$

The values of the coefficients of the differentials are found by comparison with the earlier derived formulas (32), (33) and (34). We will write formulas for computing these auxiliary values:

$$\begin{aligned}
 A &= \tan b' \cos(l'n), \\
 B &= (\alpha) \left(\frac{d\tau}{df} \right)_0 \sec b' + \tan b' \sin(l'-n) \sin I \left(\frac{db}{df} \right)_0 + \tan b' \cos(l'-n) \left(\frac{dg}{df} \right)_0, \\
 A' &= -\sin(l'-n), \\
 B' &= -\sin I \cos(l'-n) \left[\left(\frac{d\tau}{df} \right)_0 - \left(\frac{d\theta}{df} \right)_0 \right] - \sin(l'-n) \left(\frac{d\varrho}{df} \right)_0 = \\
 &= -\sin I \cos(l'-n) \left(\frac{d\tau}{df} \right)_0 + \cos(l'-n) I \left(\frac{d\theta}{df} \right)_0 - \sin(l'-n) \left(\frac{d\varrho}{df} \right)_0, \\
 C &= -\sin \beta' \cos b' \sin(\lambda'-l') g, \\
 D &= [\cos \beta' \cos b' + \sin \beta' \sin b' \cos(\lambda'-l')] g, \\
 C' &= -\cos b' \cos(\lambda'-l') g, \\
 D' &= -\sin b' \cos(\lambda'-l') g.
 \end{aligned}$$

In the last four formulas $\rho = 0.272506$.

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On the basis of geometrical considerations, we can derive individual formulas, for example, formulas for improving the crater coordinates λ, β from measurements of limb crater position angles relative to Moesting A. We use the notations: Q is the pole of the ecliptic in the lunar celestial sphere, P is the pole of the lunar equator (Figure 6).

$$\angle KQP = (l-n) - 90^\circ, \quad \angle QPK = 90^\circ - (\lambda + m - n), \quad \angle QKP = q,$$

where K is the initial crater position. From the triangle we have

$$\cos q = -\sin(\lambda + m - n) \sin(l-n) - \cos(\lambda + m - n) \cos(l-n) \cos I,$$

or

$$\cos \beta \cos q = \cos I \cos b - \sin I \sin b \sin(l-n) = (\alpha),$$

where

$$\begin{aligned}
 \cos q &= (\alpha) \sec \beta, \\
 \frac{\sin q}{\sin I} &= \frac{\cos(\lambda + m - n)}{\cos b}, \\
 \sin q &= \frac{\sin I \cos(\lambda + m - n)}{\cos b} = -\frac{\sin I \cos(l-n)}{\cos \beta} = -(\beta) \sec \beta.
 \end{aligned}$$

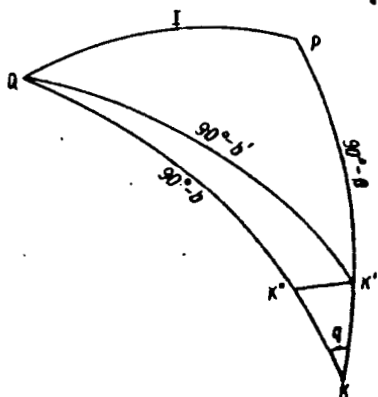


Figure 6

1. We find $db/d\beta$ under the condition that $d\lambda = 0$.

$$\begin{aligned}\angle P &= 90^\circ - (\lambda + m - n), & K(\lambda, \beta), \\ \angle Q &= (l - n) - 90^\circ, & K'(\lambda, \beta, \Delta\beta), \\ \cup KK'' &= \cup KK' \cos q, \\ \frac{db}{d\beta} &= \cos q \cdot (\lambda) \sec \beta.\end{aligned}$$

2. We find $dL/d\beta$ when $d\lambda = 0$.

$$\begin{aligned}\cup KK' &= d\beta, \\ \cup K'K'' &= d\beta \sin q \, dl \cos \delta, \\ \angle K''QK' &= dl, \\ \frac{dL}{d\beta} &= \frac{\sin q}{\cos \delta} = (\beta) \sec \delta \sec \beta.\end{aligned}$$

$\sin q$ is considered positive here if the lunar pole is situated to the west of QK (Figure 7).

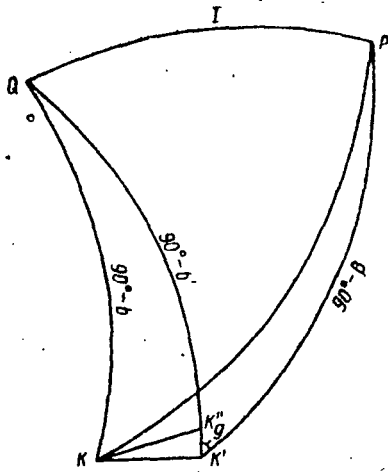


Figure 7

3. We compute $dL/d\lambda$ under the condition that $d\beta = 0$. From Figure 7 we see that

$$\begin{aligned}d\lambda &> 0, \\ \angle PK'K &= 90^\circ, \\ \angle QK'P &= q, \\ \angle QK'K &= 90^\circ - q, & dl = d\lambda \cos q \sec \delta \sec \beta, \\ \angle KQK' &= dl, \\ \angle KPK' &= d\lambda, \\ \cup PK' &= 90^\circ - \beta, \\ \cup KK' &= d\lambda \cos \beta,\end{aligned}$$

4. We compute $db/d\lambda$ under the condition that $d\beta = 0$. From Figure 7 we have:

$$\begin{aligned}OK' - OK &= d\delta, \\ K'K'' - d\delta &= -\cup K'K \sin q = -d\lambda \cos \beta \sin q, \\ d\delta &= -d\lambda \cos \beta \sin q.\end{aligned}$$

Thus,

$$\begin{aligned}\frac{dl}{d\beta} &= \frac{\sin q}{\cos b} = \frac{\sin \lambda \cos (l' - n)}{\cos \beta \cos b'} = \frac{(\beta)}{\cos \beta \cos b} = (\beta) \sec \beta \cdot \sec b' = M', \\ \frac{db}{d\beta} &= \cos q = (\alpha) \sec \beta = M', \\ \frac{dl}{d\lambda} &= \cos q \sec b' \cos \beta = (\alpha) \sec b' = N, \\ \frac{db}{d\lambda} &= -\cos \beta \sin q = -(\beta) \cdot N'.\end{aligned}$$

As a result, we have the following values for the position angle derivative for I , f , λ and β :

$$\begin{aligned}\frac{\partial \rho_{1,2}}{\partial I} &= E[(C_2 A_2 \cdot D_2 A'_2) - (C_1 A_1 \cdot D_1 A'_1)] \cdot F[(C'_2 A_2 \cdot D'_2 A'_2) - (C'_1 A_1 \cdot D'_1 A'_1)], \\ \frac{\partial \rho_{1,2}}{\partial f} &= E[(C_2 B_2 \cdot D_2 B'_2) - (C_1 B_1 \cdot D_1 B'_1)] \cdot F[(C'_2 B_2 \cdot D'_2 B'_2) - (C'_1 B_1 \cdot D'_1 B'_1)], \\ \frac{\partial \rho_{1,2}}{\partial \lambda} &= E[(C_2 N_2 \cdot D_2 N'_2) - (C_1 N_1 \cdot D_1 N'_1)] \cdot F[(C'_2 N_2 \cdot D'_2 N'_2) - (C'_1 N_1 \cdot D'_1 N'_1)], \\ \frac{\partial \rho_{1,2}}{\partial \beta} &= E[(C_2 M_2 \cdot D_2 M'_2) - (C_1 M_1 \cdot D_1 M'_1)] \cdot F[(C'_2 M_2 \cdot D'_2 M'_2) - (C'_1 M_1 \cdot D'_1 M'_1)].\end{aligned}\tag{36}$$

The values of the derivative for the distance between the crater Moesting A and the limb crater for I , f , λ and β will have the same form, but with the difference that everywhere it is necessary to replace E and F by E' and F' , for example:

$$\frac{\partial S_{1,2}}{\partial I} = E'[(C_2 A_2 \cdot D_2 A'_2) - (C_1 A_1 \cdot D_1 A'_1)] \cdot F'[(C'_2 A_2 \cdot D'_2 A'_2) - (C'_1 A_1 \cdot D'_1 A'_1)].$$

Now we can write the following equations:

$$\begin{aligned}\frac{\partial p}{\partial I} dI + \frac{\partial p}{\partial f} df + \frac{\partial p}{\partial \lambda} d\lambda + \frac{\partial p}{\partial \beta} d\beta &= p_{\text{hadm}} - p_{\text{out}} = l, \\ \frac{\partial S}{\partial I} dI + \frac{\partial S}{\partial f} df + \frac{\partial S}{\partial \lambda} d\lambda + \frac{\partial S}{\partial \beta} d\beta &= S_{\text{hadm}} - S_{\text{out}} = l'.\end{aligned}\tag{37}$$

In this monograph we give preference to the position angles method because we feel that it is a more precise observation method.

Free Libration

The position angles method makes it possible to determine free libration as well as forced libration; we intend to do this later. Using Hayn's theory, physical libration in longitude is expressed by the following equation

$$\tau = A \sin(a + t \sqrt{0.9852 M' \gamma}) - \sum \frac{M' \gamma H \sinh h}{(h')^2 - 0.9852 M' \gamma},$$

where h' is the diurnal change in the argument h , t is time in mean days, A , a are the amplitude and initial phase respectively; $M' = 3M$, M is lunar mass.

We denote $\sqrt{0.9852 M' \gamma}$ by ω . The increment l with the introduction of free libration T into the formula for τ is equal to $\Delta l = (\alpha) T \sec b$. Then in place of formula (32'), we obtain

$$\Delta l = (\alpha) T \sec b + (\alpha) (\Delta \tau - \Delta \delta) \sec b + (\delta) (\Delta I + \Delta q) \sec b.$$

The integral of physical libration in longitude can be represented in the form /102

$$\tau = A \sin a \cos \omega t + A \cos a \sin \omega t - \sum \frac{M' \gamma H \sinh h}{(h')^2 - 0.9852 M' \gamma}.$$

We use the notations

$$\begin{aligned} A \sin a &= e, & \cos \omega t &= \theta', \\ A \cos a &= \kappa, & \sin \omega t &= m'. \end{aligned}$$

Then

$$\tau = e \theta' + \kappa m' - \sum \frac{M' \gamma H \sinh h}{(h')^2 - 0.9852 M' \gamma}.$$

In this monograph free libration is assumed equal to zero. However, if its existence is taken into account, it enters into Δl in the form

$$\Delta l = \Delta \delta + (\alpha) T \sec b + (\alpha) (\Delta \tau - \Delta \delta) \sec b + (\delta) (\Delta I + \Delta q) \sec b, \quad (38)$$

where

$$T = ee' \cdot \kappa \kappa'$$

We obtained

$$\begin{aligned} dx' &= C dl \cdot D db, \\ dy' &= C' dl \cdot D' db. \end{aligned}$$

With free libration taken into account we obtain:

$$\begin{aligned} dx' &= C[dl \cdot (\alpha) T \sec \delta] \cdot D db = C(\alpha) T \sec \delta \cdot D db \cdot C(AdI + Bdf), \\ dy' &= C'[dl \cdot (\alpha) T \sec \delta] \cdot D' db = C'(\alpha) T \sec \delta \cdot D' db \cdot C'(AdI + Bdf), \\ d\rho_{1,2} &= E[C_2(\alpha_2) \sec \delta_2 - C_1(\alpha_1) \sec \delta_1] T + F[C'_2(\alpha_2) \sec \delta_2 - C'_1(\alpha_1) \sec \delta_1] T + \\ &+ \frac{\partial \rho}{\partial I} dI + \frac{\partial \rho}{\partial f} df + \frac{\partial \rho}{\partial \lambda} d\lambda + \frac{\partial \rho}{\partial \beta} d\beta. \end{aligned} \quad (39)$$

INSTRUMENT AND OBSERVATIONS

Observatory and Instrument

The observatory building with a brick facing, with an annex, in which the instrument was installed, has a circular configuration. On the inside the observatory building walls are faced with wood. Mounted atop the walls there is a metal dome with panels which can slide for opening the dome. The dome is faced on the inside with plastic. The observatory building diameter is 5 m.

A refractor, the AVR-2, made in 1960 by the Leningrad Optical-Mechanical Plant, is used for the visual observations of the moon which are made at the Main Astronomical Observatory Academy of Sciences of the Ukrainian SSR. It is mounted on an ANSh-6 parallactic mounting having a 4M-1 clock mechanism. The instrument objective is a two-lens uncemented achromat with visual correction. The objective is 20 cm in diameter and the focal length is 3 m. The instrument can also be used for taking photographs directly at the focus of the objective. The instrument is supplied with a guided eyepiece micrometer which is fitted onto a bayonet fitting. The micrometer has position rotation; the position circle is graduated each 1° .

When observing pairs of craters by the position angles method it is necessary to read the position circle with an accuracy to $1'$; accordingly, this eyepiece micrometer is not suitable for solving our problem. The Main Astronomical Observatory of the Academy of Sciences of the Ukrainian SSR has a free eyepiece micrometer which is also mounted on the tube of this instrument on a bayonet fitting. The advantage of this micrometer is that the position circle is graduated each $15'$ and by means of a vernier it is possible to make readings with an accuracy to $1'$. By means of two micrometer screws, this micrometer makes it possible to move the eyepiece relative to the reticle in two coordinates. The movements are read from a scale. The determinations which we made indicate that the barrel graduation is equal to $0''.410$, the field

of view is 16', and the eyepiece magnification is 145. The hour circle correction $\Delta t = - 3^m.5$; the collimation error is $e = - 9^s.8$.

Elements of Instrument Adjustment

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The instrument was installed in the observatory building in 1962 on a brick pillar; accordingly, it was necessary to adjust the instrument and determine the elements of its adjustment. The adjustment elements must be known for introducing reductions into the measured position angles. The circles are graduated each 15^m in right ascension and each 1° in declination. The adjustment constants were determined by the well-known differential method; this does not require circle readings. In this procedure the same star is observed at the upper and lower culminations without changing the adjustment of the tube in declination. It is also possible to observe two close stars in declination; in this case one star is observed at the upper culmination and the other at the lower culmination. By measuring the difference in declinations of these stars with the micrometer, it is possible to determine the coordinates of the instrument pole using the formula

$$\xi = \frac{1}{2}(f_1 - f_2) \cdot \frac{1}{2}(r_1 - r_2) \cdot \cos \varphi \sin \delta - \frac{1}{2}(\delta_1 - \delta_2),$$

if the instrument flexure e is known.

From similar observations of a star pair in the east and west we find the second coordinate using the formula

$$\eta = \frac{1}{2}(f_1 - f_2) \cdot \frac{1}{2}(r_1 - r_2) - \frac{1}{2}(\delta_1 - \delta_2),$$

where f is the reading on the micrometer barrel, r is refraction, δ is star declination.

We used the Hillenberg method [19], which involves the following. From a comparison of the rate of change in "instrumental declination" of a star with the rate of change (in hour angle) of the influence of refraction on star declination it is possible to obtain the adjustment constants. The rate of change in "instrumental declination" of a star was measured with the micrometer by sighting in on a star every ten minutes with a moving

wire directed along a parallel. Star observations are made with an operating clock mechanism near the circle values of the hour angles, i.e., in the south $t \sim 0^h$, in the west or in the east $t \sim \pm 6^h$, and in the north $t \sim 12^h$. For determining the coordinates of the instrument pole we have the well-known formula

$$d - (\delta + r) = \xi_1 \cos t + \eta \sin t, \quad (40)$$

where $\xi_1 = \xi - e \cos \phi \sin \delta$; d is the instrumental declination, encumbered by the error in the index of the declination circle and the value $e \sin \phi \cos \delta$; (ξ, η) are the coordinates of the instrument pole, positive to the south and west; e is tube deflection; r is refraction. Differentiating (40) for the hour angle t , we obtain the fundamental Hillenberg formula

$$\mu - g = -\xi_1 \sin t + \eta \cos t$$

Hence:

$$\begin{aligned} \text{from observations in the south} \quad \eta &= \mu - g + \xi_1 \Delta t, \\ \text{in the north} \quad \eta &= g - \mu + \xi_1 \Delta t, \\ \text{in the west} \quad \xi_1 &= g - \mu - \eta \Delta t, \\ \text{in the east} \quad \xi_1 &= \mu - g + \eta \Delta t. \end{aligned}$$

Taking into account the variation in refraction with change in temperature,

$$\delta = \delta_0 + \rho_0 \Delta t + \Delta_1 \rho + \Delta_2 \rho,$$

where ρ_0 is the rate of change in refraction at a constant temperature for a circle value of the hour angle; $\Delta_1 \rho = k_1 \Delta t$ is the variation in the rate ρ_0 for and excess (Δt) of the observed hour angle over the circle value; $\Delta_2 \rho = k_2 \Delta t$ is the variation in refraction itself with a change in temperature. The value ξ is determined from observations of stars at the hour angle $\pm 6^h$ using the formula

$$\xi_1 = k \cos \varphi \cos \sec^2 \delta \pm r k_1 \tan^2 \varphi \cos \sec^2 \delta \cos \delta \pm 1'' \Delta t \pm 0.13 \cot \delta \Delta t \pm \mu - \eta \Delta t.$$

This method was used in observing 44 stars from 1962 through 1967; as a result, we obtained $\xi = +183''$, $\eta = 0''$.

The hour angle θ of the instrumental pole and its distance from the celestial pole γ will be as follows: $\gamma = 183''$, $\theta = \arctan \frac{\eta}{\xi} = 90^\circ$; the instrument deflection is $e = + 3.3$.

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We note that the instrument adjustment elements remained stable during the entire period of observations.

Error in Position Circle Index

A very important element in investigating the instrument is the error in the position circle index, i.e., the reading at which the micrometer wire (employed in observations) is in the plane of the circle of declination. The index error was determined by two methods.

1. From observations of close star pairs in different sectors of the sky. As already mentioned, the field of view for our instrument is $16'$. Accordingly, we were extremely limited in selecting star pairs separated from each other by such a short distance. For the most part, we observed the following star pairs: Pleiades η 27, Cygnus $\alpha-e$, Ursae Minoris 24.

2. From observations of the transit of near-equatorial stars, situated not far from the moon. With a motionless tube the "working" micrometer wire was set on the diurnal motion of the star in such a way that the star did not deviate from it. In this position we made a reading on the position circle, determining declination and hour angle of the star. In a second observation the micrometer was rotated by 180° in position angle. The measured angles were corrected for differential refraction.

As an example, we give determinations of the position circle null point made by this method during the period from 22 October 1964 through 27 September 1966 (Table 9).

The data in Table 9 show that by using all the null point values determined during the period from 22 October 1964 through 28 August 1966 it is possible to obtain a reliable mean value.

In June 1964 the observations were made at the Mountain Station of the Pulkovo Observatory near Kislovodsk using the same instrument (AVR-2) as used in our observatory, but we installed our own eyepiece micrometer. We also

determined the adjustment constants for this instrument: $\xi = + 180'' \pm 4''$,
 $\eta = 0'' \pm 5''$. The position circle null point was found to equal $53^\circ 31'.2 \pm 0'.4$.

TABLE 9

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Date	Null-point	Date	Null-point	Date	Null-point	Date	Null-point
1964 X.22	112°40.0	1965 IV.15	112°41.0	1965 X.3	112°40.8	1966 I.4	112°41.5
XI.21	40.4	IV.16	41.3		40.4	II.6	42.2
XII.20	41.2	VI.11	42.2		42.1	III.8	41.4
1965 I.12	41.5	VI.15	41.4	X.4	40.4	V.4	42.2
I.13	40.8	VII.17	40.8	X.6	41.5	VI.3	40.9
I.14	41.7	VIII.18	42.2	X.10	41.8	VII.28	40.8
II.9	42.0	IX.4	42.0	XI.3	42.0		41.5
II.12	41.6	IX.6	40.7		41.8		41.3
III.16	40.6	IX.10	40.7	XI.7	42.2		40.6
III.18	41.1	IX.16	42.2				40.8

Mean 41.365 $\pm$ 0.6

As a control, individual evenings were devoted exclusively to determination of the null point by two methods. In this procedure, after determining the null point by the first method, with the instrument adjustment elements taken into account, we moved it to the place where observations were made by the second method.

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In processing position angle measurements and introducing corrections for instrument errors, we used the formula

$$p = P + k + \lambda \pm (i \sec \delta - c \tan \delta + \mu M) = P + k + \lambda \pm (\xi \cos t - \eta \sin t) \sec \delta,$$

where + corresponds to circle "precedes", - corresponds to circle "follows",
 P is the observed position angle, k is the position angle null point,
 $\lambda = \xi \sin t - \eta \cos t$, μ is coefficient of twist for the position circle index,
 $M = k n \phi \cos \delta - \cos \phi \sin \delta \cos t$. The coefficient of twist for the position circle was found from determinations to equal 0.12.

Observation Method

The use of the position angles method at the Main Astronomical Observatory of the Academy of Sciences of the Ukrainian SSR is the first experiment in studying lunar rotation without a tie-in of the crater Moesting A or other features with the lunar limbs. In this method, in addition to visual observations we also employed photographic observations. Now we will discuss processing of the lunar photographic images.

A number of studies [9-12] have already been devoted to this method. We used lunar photographs and materials from visual observations. Lunar photographs were obtained using an astrograph (objective diameter 20 cm and focal length 4.3 m) at the Astronomical Observatory of Kiev State University by A. K. Osipov during 1950-1959. The photographs were taken on *Astro-Agfa*, *Astro-Platten* and phototechnical photographic plates. On each plate there are three lunar images; one of them is situated at the geometric center of the plate (second image). Different orientation methods can be used. Orientation can be accomplished by: 1) using craters [96], 2) using images of the moon itself in its diurnal motion, 3) using the images of stars situated in any part of the sky, but at a small declination, with known instrument adjustment elements.

The photographic material which we used was oriented on stars printed on a plate from another part of the sky. When the photographs were taken the stars were situated symmetrically relative to the optical center with which the lunar center, or to be more precise, the crater Moesting A, should coincide.

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For obtaining the scale and orienting the plates during photographic work, one of the following star pairs was registered: γ , χ UMA, α and δ Del, δ , ζ Ori. The apparent coordinates of these stars were used in computing the distances between them (for scale) and the position angle (for orienting the lunar photograph). The computed position angle and the distance were encumbered by differential refraction.

Prior to measuring the lunar photograph, a number of limb craters, listed in Table 10 were noted. In selecting the craters attention was given to

their good visibility during all phases of illumination, since their configuration had to be distinct and convenient for measurement. It is also desirable that they be uniformly spaced along the lunar limb zone. It is difficult to select a crater in the southern part of the moon, and particularly in the southeastern part. In this sector there are few craters which are clearly visible and readily detectable during observations, especially visual observations.

It is desirable to select the central crater in such a way that it will lie as close as possible to the lunar center when there is zero libration in latitude and longitude. During the time of total change in libration in longitude and latitude such a crater will move approximately symmetrically relative to the lunar center.

During the time that the lunar figure and rotation was investigated we used Moesting A as the central crater. This crater is small (diameter about 5"), stands out clearly, and therefore is quickly detectable at all lunar phases (naturally, when it is situated on the illuminated part). Franz proposed that the crater Moesting A be replaced by another which in his opinion was better, i.e., Triesnecker B. The latter in actuality is situated closer to the equator and the lunar prime meridian. However, we feel that it is less convenient for observations if for no other reason than that it is difficult to detect among the nearby craters.

The lunar photographs were measured on a KIM-3 coordinate measuring instrument. From three lunar photographs present on the plate, we selected the one which was situated symmetrically relative to the imprinted star pair. This was always the middle photograph of the three. The measurements of craters were made in a rectangular coordinate system. The plate was placed in the instrument in such a way that the line connecting the star images passed along the y -axis. Then we measured the coordinates of stars for which y differed very little for both craters, whereas x differed considerably. Using the measured coordinates, we determined the position angle of this star pair. Four sightings were made on each star, using a reversion prism. Since the stars were imprinted from any part of the sky, their reduction to the position of the moon requires reduction of the position angles to the

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celestial pole from the instrument pole

$$\Delta p = (\xi \cos t - \eta \sin t) \sec \delta,$$

where ξ and η are the instrument adjustment elements, δ is the mean declination of a close star pair (Figure 8).

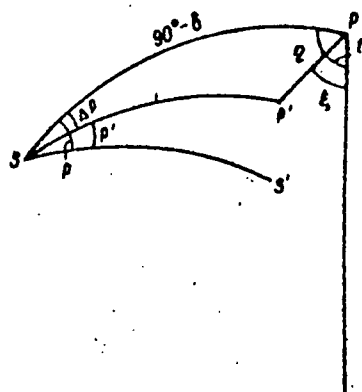


Figure 8

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$$\begin{aligned} \text{Fast } p' &= p', \\ \frac{\sin \Delta p'}{\sin \delta} &= \frac{\sin(t - \theta)}{\sin(90^\circ - \delta)}, \\ \Delta p' &= \gamma \sin(t - \theta) \sec \delta', \\ \xi &= \gamma \cos \theta, \\ \eta &= \gamma \sin \theta, \\ \Delta p' &= (\gamma \cos \theta \sin t - \gamma \sin \theta \cos t) \sec \delta', \\ \Delta p' &= (\xi \sin t - \eta \cos t) \sec \delta'. \end{aligned}$$

Now we will give the adjustment constants for the refractor at the Astronomical Observatory of Kiev State University, determined by V. K. Drofa: $\xi = -29''$, $\gamma = 38''$, $c = 0$, $\eta = +25''$, $B = 139^\circ 14'$, $l = 15''$.

The instrumental position angle of any straight line on the plate (emanating from the optical center) p' remains constant for any movement of the tube about the polar axis and in declination. It is read from the plane perpendicular to the instrument axis of declination. The true position angle p is equal to the instrumental angle plus the angle between the circle of declination and instrumental Δp .

Thus, we obtain the true position angles, free of refraction and instrumental errors, related to the true circle of declination. These position angles, determined from the star pair for the particular photograph, are the basis for a tie-in of the position angles for the directions between Moesting A and limb craters.

Simultaneously, with this same orientation of the plate, among the stars we measured the rectangular coordinates of both the crater Moesting A and the noted limb craters. The measured coordinates of the craters were used in computing the position angles of the directions between Moesting A and limb craters. A position angle error of 1' corresponds to a displacement of the crater on the lunar disk by an average of $800'' \sin 1' = \pm 0''.24$.

It therefore follows that in determining the measured position angles of limb craters relative to Moesting A it is necessary: 1) to compute the position angles of stars from the apparent coordinates in an equatorial coordinate system, 2) to determine the position angles of the directions between Moesting A and limb craters in an equatorial coordinate system using the results of measurements (Appendix, Table 3), 3) to correct these angles for differential refraction, 4) to compute the corrections for instrument adjustment, since the stars and moon at the time the photographs are taken were situated in different parts of the sky.

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Thus, we obtained the true position angles from observations.

In order to compare the observed position angles and the theoretical position angles computed using formulas (36) and related to a circle of latitude, they must be reduced to a single system (Appendix, Table 3). This requires determination of the angle between the circle of latitude and the circle of declination χ . The angle is computed from the triangle, where P is

the equatorial pole, Q is the pole of the ecliptic, ζ is the lunar center (Figure 9).

If α_{ζ} and δ_{ζ} are stipulated, then

$$\operatorname{ctan} \chi = \frac{\cos \delta}{\cos \alpha \tan \epsilon} + \sin \delta \tan \alpha, \quad (41)$$

if l and b are stipulated,

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$$\operatorname{ctan} \chi = \frac{\cos b}{\cos l \tan \epsilon} - \sin b \tan l.$$

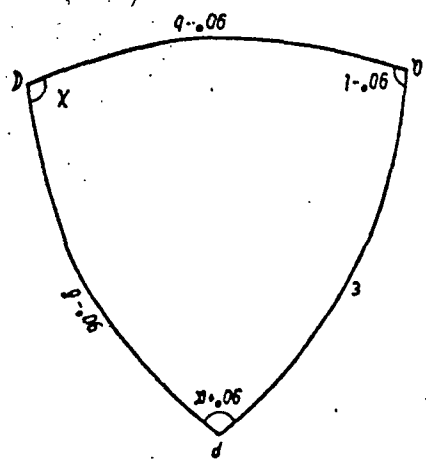


Figure 9

After having the computed and observed position angles, we can write equations in the form

$$\frac{\partial \rho}{\partial l} dl + \frac{\partial \rho}{\partial f} df + \frac{\partial \rho}{\partial \lambda} d\lambda + \frac{\partial \rho}{\partial \beta} d\beta = l, \quad (42)$$

where $l = P_{obs} - P_{comp}$, $\frac{\partial \rho}{\partial l}$, $\frac{\partial \rho}{\partial f}$, $\frac{\partial \rho}{\partial \lambda}$, $\frac{\partial \rho}{\partial \beta}$ are the computed values.

Visual Observations

Determination of the parameters of lunar rotation from photographic materials unquestionably has some advantages. These include its instantaneous nature and documentary character. The measurement of negatives can be repeated many times at leisure, repeating the measurements with different measuring instruments. In addition, photographic materials can be stored for a long time and used for different aspects of lunar research. However, this method also has shortcomings which have a negative effect on measurement accuracy: the images of features on the lunar surface are indistinct and blurred.

In our work we decided to give preference to the visual rather than the photographic method. Unquestionably, in the former case considerably more time and work is spent on observations themselves and in their processing. However, this is justified. Observing the moon visually, even taking into account atmospheric inhomogeneity and the fluctuating images, it is possible to determine the true position of features very reliably, whereas a photograph can record an individual deviation which in processing is taken to be a true image.

Beginning in September 1962 and through the present time, the author at the Main Astronomical Observatory of the Academy of Sciences of the Ukrainian SSR has been making visual observations of the moon by the position angles method; this method involves the following. As pointed out, the field of view for our instrument is 16', and therefore we set the observed craters in the field of view in such a way that the central crater Moesting A and the limb

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crater were situated at the edges of the field of view and the centers of these craters were set on the micrometer wire by micrometer rotations. The bisecting of both craters by the wire was repeatedly checked if any doubt was present. The time of setting of the wire on the center of both craters was registered using a chronometer, and the position of the wire was determined by a reading on the micrometer position circle. Similar observations were made for the next Moesting A-limb crater pair. After conclusion of observations for all the intended pairs of craters, the micrometer was rotated by 180° and observations were repeated from the last pair to the first.

If observations in both directions are made at the same speed, the average of the measurement times in forward and backward directions will be approximately identical for each pair. When processing observations the mean time of observations is the same for all pairs of craters. The Appendix, Table 1, gives the mean moments of observations in universal time.

The observations were made from the first quarter through the full moon to the last quarter, i.e., during the time when the crater Moesting A is visible. Naturally, the number of visible craters, beginning with the first quarter (western limb), increased up to the time of the full moon and then decreased to the last quarter, when it was possible to observe only craters situated in the lunar eastern hemisphere. Near-polar craters for all practical purposes could be observed from the first to the beginning of the last quarter. Thus, during a month all the limb craters could be observed.

In addition to the mentioned observations of Moesting A-limb crater, after each full observation (with very, very few exceptions) we determined the null point. We should discuss the question of what angle is measured on a photograph and what angle is measured visually. On a photograph the direction angles of the directions between Moesting A and a limb crater are measured. With respect to visual observations, in this case it is not the angle at Moesting A which is measured, but the angle at the midpoint of the straight line connecting Moesting A with the limb crater.

The theoretical values of the position angles were computed for Moesting A. Accordingly, as a comparison, the observed angles must also be computed

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for Moesting A. How does one transform from the observed position angle at the middle of the arc between craters 0 to the position angle at Moesting A?

Using the notation P for the celestial pole $\angle POK_2 = \rho_0$ for the measured position angle at the midpoint 0 (Figure 10):

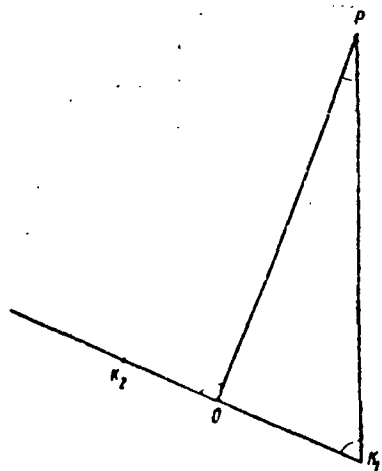


Figure 10

$\angle PK_1K_2 = \rho_M$ for the angle at Moesting A;

$$\angle OPK_1 = \frac{\alpha_1 - \alpha_2}{2} \quad ; \quad PK_1 \approx PO = \rho_0 - \delta_\alpha,$$

we will have

$$\Delta\rho = \frac{\alpha_2 - \alpha_1}{2} \sin \delta_\alpha = \frac{\Delta\alpha}{2} \sin \delta_\alpha,$$

$$\rho_M = \rho_0 + \frac{\Delta\alpha}{2} \sin \delta_\alpha.$$

This correction can be called the correction for convergence of meridians. In computing the correction for refraction it is necessary

to know the zenith distance z and the parallax angle of the moon q . Accordingly we first computed z and q for the time of observation. The correction for differential refraction in position angle was computed using the formula

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$$\Delta p = 1/2 f \tan^2 z \sin r(p-q) - f \tan z \sin q \tan \delta.$$

The parameter f is a function of temperature T , pressure B and zenith distance z :

$$f = f_0 \cdot B \cdot T,$$

where f_0 is the refraction coefficient.

Summarizing the above, we will give a list of corrections for position angles obtained from visual observations 1) for instrument adjustment,

2) for error in the position circle index, 3) for convergence of meridians, and 4) for differential refraction.

The observed position angles of limb craters at Moesting A are reckoned from a circle of declination, whereas the computed position angles are reckoned from a circle of latitude. Accordingly, as in processing of lunar photographs, it is necessary to correct visually observed position angles for the angle χ for reducing them to a circle of latitude. We thereby reduce the observed and computed position angles to a single system. Comparison of these angles (theoretical and observed) is accomplished using the same formula (41) as in the photographic method.

Selection of Craters

In determining the parameters of lunar rotation it is sufficient to select four limb craters, two near the lunar equator and two at the poles. However, this will be correct when we make observations near the full moon. However, if we begin observations approximately at the first quarter, the near-polar craters either will be entirely unobservable or they will be situated at the terminator. Thus, only the near-equatorial craters remain; they are not always of value. In this case it is better to observe craters situated along the entire apparent limb zone.

We selected a total of 26 limb craters for observation, and used Moesting A as the central crater (Table 10). We do not consider our selection of craters to be final. The fact is that this problem requires great attention, preparation, and experience in observation. Those limb craters which have approximately the same size as Moesting A will be best. In this case the observation errors caused by phase will be minimal. There are not too many craters with a distinct configuration near the limb. It is easiest to select such craters in the northern part, and most difficult to select such craters in the southern part, particularly in the southeast.

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In Figure 11 we show a diagram of the arrangement of the craters which we observed. The numbers in this figure correspond to those in Table 10. As can be seen from Figure 12 a, the near-equatorial craters can always be reliably observed from the first to the last quarter, since bisecting with the

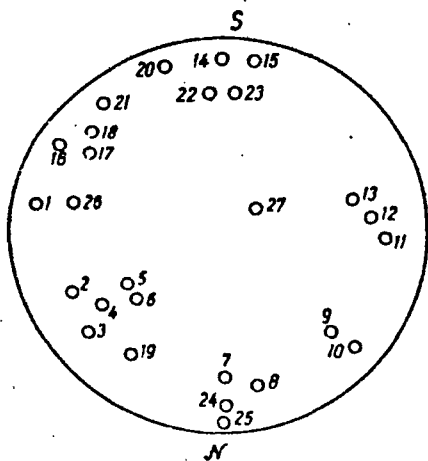


Figure 11

wire is done in two symmetrical parts. In this case the crater phase does not introduce any error into the measured angles. With respect to near-polar craters, in this case an error can be introduced (Figure 12 b). This influence was investigated by the author and D. P. Duma; he rendered considerable assistance in the observations. In the investigations we measured the position angles of the directions for Moesting A (1) -- limb crater (2) (Figure 13) in the following sequence:

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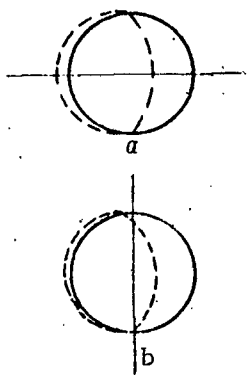


Figure 12

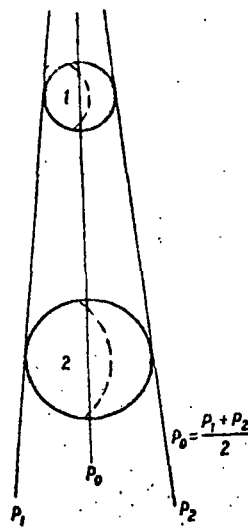


Figure 13

- 1) the micrometer wire is set at a tangent to both craters and a micrometer reading (p_1) is made; 2) both craters are bisected with the wire (p_0) and
- 3) the wire is set at a tangent to both craters from the other side (p_2).

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If $1/2 (p_1 + p_2) = p_0$, the influence of phase does not introduce errors into the measured angles. In actuality, no such error was discovered in visual observation. This can be attributed to the fact that in visual observations the illuminated and dark edges of the crater can be clearly observed. We used limb craters of different sizes.

CHAPTER VI

DETERMINATION OF PARAMETERS OF LUNAR ROTATION

Writing of Normal Equations

Conditional equations in the following form can be written for each Moesting A-limb crater direction

$$\frac{\partial l}{\partial l} dl + \frac{\partial l}{\partial f} df + \frac{\partial l}{\partial \lambda} d\lambda + \frac{\partial l}{\partial \beta} d\beta = l.$$

In these equations the coefficients of the unknowns were computed using formula (36), whereas the l value was found from Table 3 (Appendix), as $l = p_{obs} + x - p_{comp}$. The conditional equations for each Moesting A-limb crater direction were written separately for each limb crater. Since we observed 26 limb craters, we derived 26 groups, a separate group for each limb crater. We will not give separately the conditional equations for each Moesting A-limb crater direction, but will give the normal equations already written for them in the form

$$\begin{aligned} [aa]dl + [ab]df + [ac]d\lambda + [ad]d\beta &= [al], \\ [bb]df + [bc]d\lambda + [bd]d\beta &= [bl], \\ [cc]d\lambda + [cd]d\beta &= [cl], \\ [dd]d\beta &= [dl]. \end{aligned}$$

Here we introduce the following notations:

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$$\frac{\partial l}{\partial l} = a, \quad \frac{\partial l}{\partial f} = b, \quad \frac{\partial l}{\partial \lambda} = c, \quad \frac{\partial l}{\partial \beta} = d, \quad p = p_{obs} - p_{comp}.$$

The number of conditional equations for each Moesting A-limb crater direction (see Table 10) is: for 1 - 134; 2 - 183; 3 - 21; 4 - 304; 5 - 240; 6 - 241; 7 - 237; 8 - 230; 9 - 75; 10 - 60; 11 - 198; 12 - 185; 13 - 195; 14 - 42; 15 - 42; 16 - 74; 17 - 27; 18 - 25; 19 - 161; 20 - 74; 21 - 83; 22 - 78; 23 - 13; 24 - 24; 25 - 28; 26 - 164. A total of 3,138 conditional equations were written.

TABLE 10

Crater Number Name	α	β
1. Lapeyrouse A	+74 ⁰ 59 00"	- 9 ⁰ 14 00"
2. Pocard	+54 49 30	+14 36 00
3. Peirce	+53 31 30	+18 19 12
4. Proclus	+46 48 30	+16 04 12
5. Macrobius A	+40 17 36	+19 30 54
6. Macrobius B	+40 46 00	+20 52 54
7. Eudoxus A	+20 05 06	+45 42 12
8. Egede A	+10 30 24	+51 33 06
9. Maupertius A	-24 42 30	+50 42 36
10. Gondamine A	-30 04 30	+54 23 48
11. Lohrmann A	-62 38 42	- 0 44 12
12. Grimaldi C	-61 24 06	- 2 36 00
13. Damoiseau E	-58 18 06	- 5 12 18
14. Clavius e	-14 05 27	-57 35 48
15. Clavius d	-17 58 31	-57 59 34
16. Furnerius A	+59 09 00	-33 37 00
17. Sterius A	+1 40 30	-31 47 30
18. Fraunhofer G	+58 27 18	-38 29 18
19. Tralles A	+46 54 42	+27 22 12
20. Zach	+ 6 46 00	-61 08 00
21. Janssen k	+42 12 42	-46 07 42
22. Deluc	- 6 54 00	-54 14 00
23. Maginus	-10 03 09	-52 30 01
24. Calle	+22 15 42	+55 37 24
25. Calle c	+24 34 06	+57 48 42
26. Langrenus	+66 16 18	- 9 46 24
27. Moesting A	- 5 10 42	- 3 11 42

Using the conditional equations, we write normal equations separately for each direction.

$$\begin{array}{rcll}
 108.528 \, dI & + & 29.420 \, df & + & 9.233 \, d\lambda & - & 0.612 \, d\beta & = & -25.329 \\
 398.080 & & & + & 5.973 & & -0.400 & = & -40.267 \\
 & & & + & 2.075 & & -0.122 & = & -1.544 \\
 & & & & & & 0.009 & = & 0.076 \\
 \\
 115.113 & + & 12.061 & + & 0.794 & + & 0.221 & = & -46.869 \\
 273.631 & & & + & 20.449 & + & 5.054 & = & +1.255 \\
 & & & & 7.929 & + & 5.863 & = & +1.806 \\
 & & & & & & 0.517 & = & +0.744
 \end{array}$$

3					
14.977	+ 4.099	- 0.387	- 0.136	= +	9.971
	18.226	- 0.956	- 0.210	= +	2.140
		0.480	+ 0.131	= -	0.162
			0.037	= -	0.054
4					
181.432	+12.900	+ 1.910	+ 0.995	= -	29.772
	345.878	+18.692	+ 6.817	= +	23.056
		11.131	+ 3.595	= +	48.851
			1.376	= +	16.304
5					
146.155	+ 6.973	+ 1.330	- 0.831	= -	29.532
	238.806	+10.698	+ 4.951	= -	15.673
		9.822	+ 4.151	= +	9.085
			1.772	= +	3.514
6					
143.541 <i>dI</i>	+ 10.292 <i>df</i>	+ 2.352 <i>dλ</i>	+ 0.371 <i>dρ</i>	= -	6.396
	246.169	+13.333	+ 6.488	= +	8.878
		11.065	+ 4.863	= +	0.203
			2.165	= -	0.087
7					
180.632	+ 33.362	- 6.977	- 2.674	= +	9.525
	378.032	+19.221	+11.558	= -	30.516
		31.707	+12.249	= -	20.639
			5.797	= -	2.000
8					
127.859	+ 26.900	- 7.470	- 0.922	= +	10.006
	443.200	+28.909	+ 7.425	= -	31.253
		41.434	+ 6.197	= -	11.219
			3.254	= +	1.171
9					
47.289	+ 18.498	+ 3.086	- 0.247	= +	11.860
	180.103	+28.404	-18.738	= -	0.977
		17.085	-10.946	= -	0.966
			7.433	= +	1.122
10					
34.264	+ 22.625	+ 1.319	- 1.053	= -	1.892
	146.274	+23.344	-15.019	= +	1.720
		16.928	-11.181	= -	3.748
			7.887	= +	3.897

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$$\begin{array}{rcllcl}
 & & 11 & & \\
 137.697 dI & - 34.947 df & - 11.431 d\lambda & + 0.441 d\beta & = +90.650 \\
 & 667.416 & + 12.861 & = 0.655 & = -14.417 \\
 & & 4.100 & = 0.156 & = - 8.972 \\
 & & & 0.008 & = - 0.328
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 12 & & \\
 134.970 & - 8.857 & - 9.454 & + 0.258 & = +87.904 \\
 & 622.002 & + 8.795 & + 0.300 & = -17.905 \\
 & & 2.916 & + 0.180 & = - 9.108 \\
 & & & 0.013 & = - 0.390
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 13 & & \\
 133.844 dI & - 28.000 df & - 9.375 d\lambda & - 0.340 d\beta & = +118.600 \\
 & 557.649 & + 4.860 & + 0.008 & = - 9.296 \\
 & & 2.775 & 0.088 & = - 7.620 \\
 & & & 0.006 & = - 0.173
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 14 & & \\
 21.452 & - 5.323 & + 0.373 & + 0.114 & = + 13.591 \\
 & 145.315 & - 3.127 & - 0.348 & = - 11.642 \\
 & & 14.949 & + 6.166 & = + 14.100 \\
 & & & 3.194 & = + 5.413
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 15 & & \\
 21.982 & - 4.005 & + 1.047 & + 0.223 & = + 8.729 \\
 & 150.062 & - 3.955 & - 0.841 & = - 2.935 \\
 & & 15.619 & + 7.349 & = - 4.248 \\
 & & & 4.059 & = - 1.112
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 16 & & \\
 55.935 & - 11.512 & - 1.347 & + 0.229 & = + 19.004 \\
 & 156.376 & - 5.583 & + 2.192 & = + 1.645 \\
 & & 5.635 & - 3.006 & = - 21.027 \\
 & & & 0.733 & = + 7.263
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 17 & & \\
 17.447 & - 1.352 & - 1.937 & + 0.600 & = - 0.712 \\
 & 21.315 & + 1.411 & - 0.512 & = - 0.943 \\
 & & 1.325 & - 0.460 & = + 0.532 \\
 & & & 0.163 & = - 0.042
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 18 & & \\
 21.740 & - 0.614 & - 3.570 & + 1.348 & = + 33.052 \\
 & 34.664 & + 1.187 & - 0.442 & = - 2.700 \\
 & & 8.845 & - 1.139 & = - 9.204 \\
 & & & 0.461 & = + 3.277
 \end{array}$$

$$\begin{array}{rcllcl}
 & & 19 & & \\
 106.845 & - 4.940 & - 1.390 & - 0.347 & = - 41.930 \\
 & 192.695 & + 34.577 & + 15.748 & = + 7.340 \\
 & & 13.281 & + 6.200 & = + 6.443 \\
 & & & 2.917 & = + 3.722
 \end{array}$$

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$$\begin{array}{rclclcl}
 & & 20 & & & \\
 40.367 dI & -43.385 df & + 5.649 d\lambda & - 0.286 d\beta & = & + 1.501 \\
 & 270.484 & +19.420 & - 3.634 & = & + 78.899 \\
 & & 29.148 & - 0.353 & = & + 32.304 \\
 & & & 0.600 & = & + 0.104
 \end{array}$$

$$\begin{array}{rclclcl}
 & & 21 & & & \\
 57.304 & -45.976 & - 3.282 & + 1.144 & = & + 5.841 \\
 & 129.307 & - 7.602 & + 4.604 & = & + 0.832 \\
 & & 12.776 & - 5.580 & = & + 6.594 \\
 & & & 2.508 & = & - 2.734
 \end{array}$$

$$\begin{array}{rclclcl}
 & & 22 & & & \\
 49.486 & -39.936 & + 4.339 & + 3.510 & = & + 4.320 \\
 & 212.410 & + 3.019 & - 0.012 & = & + 9.341 \\
 & & 24.110 & + 6.833 & = & - 1.209 \\
 & & & 2.838 & = & - 0.147
 \end{array}$$

$$\begin{array}{rclclcl}
 & & 23 & & & \\
 6.037 & - 8.343 & + 2.158 & + 0.450 & = & + 0.301 \\
 & 36.176 & + 2.400 & + 0.984 & = & - 5.429 \\
 & & 3.949 & + 0.973 & = & + 0.124 \\
 & & & 0.292 & = & - 0.725
 \end{array}$$

$$\begin{array}{rclclcl}
 & & 24 & & & \\
 16.245 & +12.835 & + 3.201 & + 0.606 & = & + 3.685 \\
 & 55.770 & + 5.421 & + 0.173 & = & + 7.404 \\
 & & 6.151 & + 7.959 & = & + 2.415 \\
 & & & 0.716 & = & - 0.314
 \end{array}$$

$$\begin{array}{rclclcl}
 & & 25 & & & \\
 21.684 & + 7.758 & + 1.265 & + 0.072 & = & + 7.585 \\
 & 47.585 & + 5.048 & - 0.041 & = & + 1.365 \\
 & & 6.266 & + 1.930 & = & + 4.533 \\
 & & & 0.746 & = & + 1.579
 \end{array}$$

$$\begin{array}{rclclcl}
 & & 26 & & & \\
 101.707 dI & +34.074 df & + 7.619 d\lambda & - 0.637 d\beta & = & - 1.247 \\
 & 344.304 & + 4.969 & - 0.521 & = & + 20.332 \\
 & & 2.102 & - 0.177 & = & + 0.831 \\
 & & & 0.019 & = & + 0.017
 \end{array}$$

In solving these 26 equations with four unknowns, we obtained the following values, where ϵ is the mean square error in one conditional equation with weight one.

$$\begin{aligned}
1 \\
\Delta J &= -17.4 \pm 19'' \\
\Delta f &= -0.108 \pm 0.12 \\
\Delta \lambda &= -18.5 \pm 4.34'' \\
\Delta \beta &= -19.34'' \pm 17'' \\
\varepsilon &= \pm 2.2
\end{aligned}$$

$$\begin{aligned}
2 \\
\Delta J &= -25'' \pm 10'' \\
\Delta f &= +0.105 \pm 0.118 \\
\Delta \lambda &= +7'' \pm 2'' \\
\Delta \beta &= +11'' \pm 51'' \\
\varepsilon &= \pm 1.9
\end{aligned}$$

$$\begin{aligned}
3 \\
\Delta J &= +34'' \pm 32'' \\
\Delta f &= -0.048 \pm 0.54 \\
\Delta \lambda &= -1.45'' \pm 5.00'' \\
\Delta \beta &= +7.11'' \pm 3.28'' \\
\varepsilon &= \pm 1.5
\end{aligned}$$

$$\begin{aligned}
4 \\
\Delta J &= -9'' \pm 10'' \\
\Delta f &= -0.166 \pm 0.11 \\
\Delta \lambda &= +4.46'' \pm 1.20'' \\
\Delta \beta &= +1.25'' \pm 5.1'' \\
\varepsilon &= \pm 2.32
\end{aligned}$$

$$\begin{aligned}
9 \\
\Delta J &= +15'' \pm 22'' \\
\Delta f &= 0.021 \pm 0.20 \\
\Delta \lambda &= 0'' \pm 2.31'' \\
\Delta \beta &= +6'' \pm 3.52'' \\
\varepsilon &= \pm 2.28
\end{aligned}$$

$$\begin{aligned}
10 \\
\Delta J &= -2'' \pm 32'' \\
\Delta f &= +0.018 \pm 0.22 \\
\Delta \lambda &= +3.53'' \pm 3.10'' \\
\Delta \beta &= +6.1'' \pm 4.55'' \\
\varepsilon &= \pm 2.28
\end{aligned}$$

$$\begin{aligned}
11 \\
\Delta J &= +45'' \pm 14'' \\
\Delta f &= +0.013 \pm 0.10 \\
\Delta \lambda &= +5.21'' \pm 39.16'' \\
\Delta \beta &= +1.048.49'' \pm 1.010.46'' \\
\varepsilon &= \pm 2.38
\end{aligned}$$

$$\begin{aligned}
5 \\
\Delta J &= -14'' \pm 11'' \\
\Delta f &= -0.066 \pm 0.15 \\
\Delta \lambda &= +9.20'' \pm 7.20'' \\
\Delta \beta &= -19.49'' \pm 16.55'' \\
\varepsilon &= \pm 2.20
\end{aligned}$$

$$\begin{aligned}
6 \\
\Delta J &= -6'' \pm 12'' \\
\Delta f &= +0.068 \pm 0.15 \\
\Delta \lambda &= +4.23'' \pm 2.4'' \\
\Delta \beta &= -10.5'' \pm 14.50'' \\
\varepsilon &= \pm 2.25
\end{aligned}$$

$$\begin{aligned}
7 \\
\Delta J &= +4'' \pm 12'' \\
\Delta f &= -0.125 \pm 0.13 \\
\Delta \lambda &= -2.55'' \pm 59'' \\
\Delta \beta &= +6.6'' \pm 2.20'' \\
\varepsilon &= \pm 2.36
\end{aligned}$$

$$\begin{aligned}
8 \\
\Delta J &= +5'' \pm 14'' \\
\Delta f &= -0.071 \pm 0.12 \\
\Delta \lambda &= -25'' \pm 29'' \\
\Delta \beta &= +1.20'' \pm 1.14'' \\
\varepsilon &= \pm 2.54
\end{aligned}$$

$$\begin{aligned}
15 \\
\Delta J &= +27'' \pm 29'' \\
\Delta f &= -0.027 \pm 0.19 \\
\Delta \lambda &= -1.12'' \pm 1.30'' \\
\Delta \beta &= +1.43'' \pm 2.55'' \\
\varepsilon &= \pm 2.25
\end{aligned}$$

$$\begin{aligned}
16 \\
\Delta J &= +16'' \pm 22'' \\
\Delta f &= -0.079 \pm 0.22 \\
\Delta \lambda &= -1.21'' \pm 57'' \\
\Delta \beta &= +4.21'' \pm 2.36'' \\
\varepsilon &= \pm 2.68
\end{aligned}$$

$$\begin{aligned}
17 \\
\Delta J &= +15'' \pm 47'' \\
\Delta f &= -0.032 \pm 0.63 \\
\Delta \lambda &= +17.40'' \pm 5.1'' \\
\Delta \beta &= +48.24'' \pm 43.59'' \\
\varepsilon &= \pm 2.82
\end{aligned}$$

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$$\begin{aligned}
 &12 \\
 \Delta J &= +34'' \pm 14'' \\
 \Delta f &= -0.002 \pm 0.10 \\
 \Delta \lambda &= +0'30'' \pm 15'10'' \\
 \Delta \beta &= 24'' \pm 54'0'' \\
 \epsilon &= \pm 2'32
 \end{aligned}$$

$$\begin{aligned}
 &13 \\
 \Delta J &= +57'' \pm 14'' \\
 \Delta f &= +0.033 \pm 0.10 \\
 \Delta \lambda &= -16'' \pm 1'40'' \\
 \Delta \beta &= +30'0'' \pm 34'0'' \\
 \epsilon &= \pm 2'37
 \end{aligned}$$

$$\begin{aligned}
 &14 \\
 \Delta J &= +37'' \pm 26'' \\
 \Delta f &= -0.035 \pm 0.17 \\
 \Delta \lambda &= +1'8'' \pm 1'9'' \\
 \Delta \beta &= -32'' \pm 2'29'' \\
 \epsilon &= \pm 1'99
 \end{aligned}$$

$$\begin{aligned}
 &21 \\
 \Delta J &= +16'' \pm 25'' \\
 \Delta f &= +0.132 \pm 0.29 \\
 \Delta \lambda &= +52'' \pm 4'05'' \\
 \Delta \beta &= +29'' \pm 9'26'' \\
 \epsilon &= \pm 2'45
 \end{aligned}$$

$$\begin{aligned}
 &22 \\
 \Delta J &= +10'' \pm 23'' \\
 \Delta f &= +0.075 \pm 0.17 \\
 \Delta \lambda &= -3'' \pm 51'' \\
 \Delta \beta &= -7'' \pm 2'37'' \\
 \epsilon &= \pm 2'28
 \end{aligned}$$

$$\begin{aligned}
 &23 \\
 \Delta J &= -17'' \pm 1'10'' \\
 \Delta f &= -0.066 \pm 0.58 \\
 \Delta \lambda &= +3'45'' \pm 3'4'' \\
 \Delta \beta &= -14'3'' \pm 11'8'' \\
 \epsilon &= \pm 1'11
 \end{aligned}$$

$$\begin{aligned}
 &18 \\
 \Delta J &= +54'' \pm 34'' \\
 \Delta f &= +0.080 \pm 0.39 \\
 \Delta \lambda &= -30'33'' \pm 15'7'' \\
 \Delta \beta &= -1'11'0'' \pm 34'24'' \\
 \epsilon &= \pm 2'27
 \end{aligned}$$

$$\begin{aligned}
 &19 \\
 \Delta J &= -30'' \pm 13'' \\
 \Delta f &= +0.054 \pm 0.23 \\
 \Delta \lambda &= -18'11'' \pm 45'' \\
 \Delta \beta &= +39'35'' \pm 19'36'' \\
 \epsilon &= 2'24
 \end{aligned}$$

$$\begin{aligned}
 &20 \\
 \Delta J &= -6'' \pm 23'' \\
 \Delta f &= +0.019 \pm 0.16 \\
 \Delta \lambda &= +1'7'' \pm 25'' \\
 \Delta \beta &= +33'' \pm 2'52'' \\
 \epsilon &= \pm 2'08
 \end{aligned}$$

$$\begin{aligned}
 &24 \\
 \Delta J &= +4'' \pm 42'' \\
 \Delta f &= +0.059 \pm 0.39 \\
 \Delta \lambda &= +36'' \pm 2'30'' \\
 \Delta \beta &= -14'' \pm 7'0'' \\
 \epsilon &= \pm 2'45
 \end{aligned}$$

$$\begin{aligned}
 &25 \\
 \Delta J &= +21'' \pm 24'' \\
 \Delta f &= -0.058 \pm 0.36 \\
 \Delta \lambda &= +17'' \pm 2'10'' \\
 \Delta \beta &= +1'20'' \pm 6'8'' \\
 \epsilon &= \pm 1'84
 \end{aligned}$$

$$\begin{aligned}
 &26 \\
 \Delta J &= -4'' \pm 14'' \\
 \Delta f &= +0.066 \pm 0.11 \\
 \Delta \lambda &= +2'27'' \pm 3'7'' \\
 \Delta \beta &= +23'6'' \pm 31'4'' \\
 \epsilon &= \pm 2'05
 \end{aligned}$$

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We obtained 26 values of the sought-for parameters dI and df . In addition we obtained the corrections to the selenographic coordinates $d\lambda$ and $d\beta$. These values are not simply corrections to the selenographic coordinates of limb craters, but are corrections to the limb crater-Moesting A differences. If

the coordinates of the crater Moesting A are considered precise, these corrections can be used as corrections for the limb craters whose coordinates are given in Table 10. The corrections for the constant parameters of lunar rotation dI and df were determined from all 26 normal equations with their weights taken into account. As a result of this combining, we obtained the following final values for corrections of the parameters:

$$\begin{aligned} dJ &= + 4'' \pm 3'', \\ df &= - 0.022 \pm 0.030. \end{aligned}$$

Taking into account the initial values $I_0 = 1^{\circ}33'00''$ and $f_0 = 0.85$, we obtain:

$$\begin{aligned} J &= 1^{\circ}33'04'' \pm 3'', \\ f &= 0.828 \pm 0.030. \end{aligned}$$

Assuming the selenographic coordinates of the craters to be precise, it is possible to determine dI and df . In this procedure, after summing the coefficients of dI and df from all 26 equations, we obtain the following normal equations:

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$$\begin{aligned} 1994.592 dJ - 5.822 df &= 252.443, \\ -5.822 dJ + 6309.781 df &= -69.746. \end{aligned}$$

The number of equations is 3,138. Solution of the normal equation gives

$$\begin{aligned} \Delta J &= + 8'' \pm 8'', \\ \Delta f &= - 0.11 \pm 0.029. \end{aligned}$$

The error in one conditional equation is $\epsilon = \pm 2.24$. The final values of the sought-for parameters are

$$\begin{aligned} J &= 1^{\circ}33'08'' \pm 8'', \\ f &= 0.839 \pm 0.029. \end{aligned}$$

Accordingly, it can be seen that both results virtually coincide. The difference in I (4'') and in f (0.01) are within the limits of error.

Discussion of Results

At the present time there is no unanimous opinion concerning the values of the parameters I and f . Researchers concerned with physical libration have concentrated their main attention on determining f . Investigations have yielded two values for this parameter on either side of a critical value. The question as to which value should be preferred still remains open. Many researchers firmly adhere to the opinion that $f = 0.60$. The Polish astronomer Koziel developed a computation method which eliminates the separation of the processing procedure into two stages. He feels that the solution will be correct if the sought-for parameters are introduced into the initial observation equations

$$-\sin \pi_i' \sec \pi_i'' (\delta \Delta \pi) - \cos \pi_i' \sec \pi_i'' (\Delta \delta) + \sec \pi_i'' (\Delta R_{\odot}^i) = \delta_0 - \delta_{\alpha},$$

where $\pi_i' = \rho_i - \pi_i' - \pi_i''$.

The unknowns are directly related to the measured parameters, and not to functions of unknowns, as in the Bessel-Wichmann and Hayn methods.

The position angles method is free of these shortcomings. In this method /129 the measurement of the position angle for the Moesting A-limb crater direction is processed independently.

Until now the processing of photographic and visual observations has been done by the position angles method with determination only of the constants of lunar physical libration ΔI and Δf . This has been mentioned as a shortcoming. It has been felt that the selenographic coordinates of the crater Moesting A, as well as the coordinates for limb craters which we used in processing observations, can contain errors which introduce inaccuracies into the determinations of the parameters of lunar rotation. We have taken this comment into account and introduced the additional unknowns $d\lambda$ and $d\beta$. However, in actuality this had little effect on the results of determination of dI and df . We feel that the most important errors in the I and f values may be introduced not by errors in the selenographic coordinates, but by the elevations of the craters which we used, which were not taken into account. In the future it will be desirable to take into account corrections for the

elevation of craters. It is desirable to perform the processing by another method, which involves the following. Adding to the first equation (36) the unknown $d\rho$, we obtain:

$$\begin{aligned} & \left(\frac{\partial \rho_{12}}{\partial \lambda} - \frac{\partial \rho_{13}}{\partial \lambda} \right) d\lambda + \left(\frac{\partial \rho_{12}}{\partial \beta} - \frac{\partial \rho_{13}}{\partial \beta} \right) d\beta + \left(\frac{\partial \rho_{12}}{\partial \lambda} - \frac{\partial \rho_{13}}{\partial \lambda} \right) d\lambda + \\ & + \left(\frac{\partial \rho_{12}}{\partial \beta} - \frac{\partial \rho_{13}}{\partial \beta} \right) d\beta + \left(\frac{\partial \rho_{12}}{\partial \lambda} - \frac{\partial \rho_{13}}{\partial \lambda} \right) d\lambda + \left(\frac{\partial \rho_{12}}{\partial \beta} - \frac{\partial \rho_{13}}{\partial \beta} \right) d\beta = 0 \end{aligned}$$

where 1,2 is the Moesting A-limb crater 2, direction 1,3 is the Moesting A-limb crater 3, direction etc. Thus, we can take the difference for all possible combinations. These differences can exclude all instrumental (systematic) errors introduced during observations. For example, having the difference of two directions Moesting A-limb crater, differing from one another by not less than 80° in position angle for a particular observation, we can then take the difference in these differences. Essentially, the angle between two directions will change its value appreciably as a result of lunar physical libration. For example, we will take two near-equatorial limb craters or two near-polar limb craters.

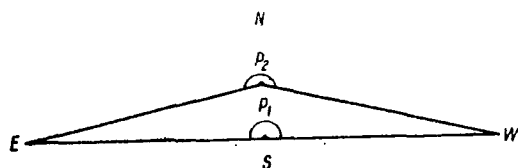


Figure 14

Figure 14 shows that if we observe position angles of directions from Moesting A to two near-equatorial craters (eastern and western), the angle between them will change appreciably with a change of libration in latitude, but will be constant with a change in libration in longitude.

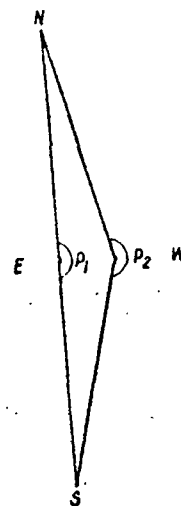


Figure 15

However, if we take the angle between two directions from Moesting A to two near-polar craters, northern and southern (Figure 15), in this case the picture will be the reverse: the angle changes with a change in libration in longitude. We cited an example for different directions for two evenings. However, a considerable number of such angles can be observed. If we take the differences for the same angle observed on different evenings, we will thereby exclude the systematic errors introduced into the position angles by inexact knowledge of the selenographic coordinates of both the limb craters and Moesting A, as well as the same radius vector $\rho = 0.272506$ which we erroneously adopted. Later this shortcoming can be eliminated in the following way. Denoting the expression in (43) for dI by a' , for df by b' , for $d\lambda$ by c' , for $d\beta$ by d' , and for $d\rho$ by e' , we obtain:

$$(a'_1 \ a'_2) dI + (b'_1 \ b'_2) df + (c'_1 \ c'_2) d\lambda + (d'_1 \ d'_2) d\beta + (e'_1 \ e'_2) d\rho = l_1 \ l_2.$$

Also taking the differences for different evenings, we thereby reduce the coefficients of $d\lambda$, $d\beta$ and $d\rho$ to zero. Only the coefficients of principal parameters of lunar rotation dI and df remain. Solving the equations separately for each angle, we find dI and df values free of the influence of selenographic coordinates and the elevations of craters.

We feel that this has little effect on the dI and df values. The position angles method which we employed is the first attempt at doing away with total reliance on the lunar limb zone in determining the parameters of lunar rotation. The method unquestionably requires improvement and the continuation of observations. In this study the results were based jointly on photographic and visual observations. It is desirable to collect observational data over a long period only by visual observations.

The values of the constants of lunar rotation presented in this study must be considered preliminary because they were obtained for an initial value $f_0 = 0.85$. In order to obtain a more objective evaluation of the determined parameters, the author is processing these observations for an initial value $f_0 = 0.60$ and is determining the wave $a_3 \sin g'$. The results of this processing will be published in the near future.

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A P P E N D I X

TABLE 1

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No	Date	UT	α'	δ'	h'	λ'	β'
1	1950.II.29	18 ^h 59 ^m 32 ^s	8 ^h 48 ^m 35 ^s .07	21 ^o 52'39"	15'53".9	20 ^h 34 ^m 28 ^s .67	-3 ^o 54'12"
2	IV.25	18 15 52	9 10 12.87	20 13 50	16 16.2	20 55 44.87	-3 44 30
3	27	18 21 21	10 33 45.13	11 21 10	16 25.1	22 23 23.27	-2 07 59
4	27	19 10 46	10 34 57.20	11 08 17	16 23.7	22 24 48.07	-2 02 34
5	27	20 45 33	10 35 45.40	10 59 47	16 24.2	22 25 44.67	-1 59 04
6	30	20 10 42	13 13 24.13	- 9 52 58	16 57.0	1 22 41.73	1 57 04
7	V.24	19 50 54	10 19 16.40	12 40 58	16 02.4	22 08 15.13	-2 04 38
8	26	18 42 36	11 56 36.47	0 16 07	16 29.7	23 56 27.60	0 05 28
9	26	19 28 51	11 57 40.60	- 0 01 52	16 28.7	23 57 55.13	0 15 35
10	28	20 21 19	13 44 05.33	-13 52 53	16 47.4	1 56 32.27	2 54 04
11	VI.29	21 27 30	18 34 17.73	-29 05 16	16 28.1	6 33 36.53	5 55 08
12	VII. 2	23 16 31	21 42 30.20	-18 08 54	15 54.9	9 27 23.33	4 08 40
13	3	23 09 28	22 32 56.00	-12 31 55	15 39.9	10 20 53.67	3 09 16
14	3	23 19 58	22 32 50.60	-12 32 08	15 40.2	10 20 48.47	3 08 58
15	5	23 30 58	23 56 27.00	- 0 35 13	15 16.5	11 55 48.53	0 11 07
16	23	19 16 50	15 02 52.13	-21 45 34	16 20.5	3 17 44.47	4 19 54
17	27	20 42 39	19 20 14.07	-27 59 33	16 16.5	7 11 53.27	5 46 15
18	1950.VII.30	22 ^h 32 ^m 42 ^s	22 ^h 12 ^m 37 ^s .87	-14 ^o 52'04"	15'51".3	9 ^h 59 ^m 19 ^s .60	3 ^o 23'03"
19	31	23 10 39	23 01 27.93	- 8 40 54	15 41.5	10 52 42.67	2 14 28
20	VIII. 7	0 47 54	3 36 50.53	23 13 23	15 04.9	15 49 39.27	-3 44 03
21	25	18 49 15	20 57 50.53	-22 11 10	14 54.6	8 42 25.00	4 47 15
22	27	20 14 34	22 41 39.80	-11 12 07	15 41.1	10 30 47.93	2 42 51
23	28	19 27 00	23 23 41.13	- 5 46 33	15 30.8	11 17 33.20	1 42 40
24	29	23 08 37	0 11 11.30	0 47 30	15 37.0	12 11 31.30	0 23 09
25	30	22 24 00	0 57 10.13	7 03 59	15 20.6	13 3 33.00	-0 52 52
26	IX.22	20 20 22	21 32 17.80	-18 55 06	15 45.4	9 17 12.53	4 05 20
27	26	1 59 31	0 43 01.53	5 27 22	15 22.0	12 48 05.47	-0 45 45
28	29	1 30 25	2 15 47.67	16 43 35	15 12.1	14 29 11.73	-2 56 10
29	X. 2	0 44 55	5 38 12.80	28 13 41	15 13.1	17 40 44.47	-4 52 17
30	4	1 59 09	6 35 21.87	28 01 53	15 20.7	18 31 18.00	-4 49 25
31	28	20 13 30	0 29 01.67	3 26 60	15 20.1	12 32 06.27	-0 17 18
32	XI. 1	1 59 02	7 12 11.47	26 58 04	15 20.3	19 04 19.27	-4 31 20
33	2	4 13 57	8 09 36.40	24 09 21	15 30.2	19 57 23.20	-3 56 59
34	XII.31	0 44 35	11 39 40.07	1 24 21	15 52.7	23 39 06.53	0 43 47

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
35	1951. I.26	23 ^h 58 ^m 05 ^s	11 ^h 30 ^m 38 ^s .60	2 ^o 52'19"	15'52".2	23 ^h 28 ^m 30 ^s .33	0 ^o 16'30"
36	II.15	22 14 00	5 06 35.47	27 38 17	15 01.6	17 12 36.87	-4 44 17
37	IV.16	19 54 04	9 39 02.80	15 50 03	15 38.5	21 27 17.60	-1 41 15
38	IX.17	20 06 50	10 25 38.60	10 15 08	15 52.4	22 15 35.87	-0 22 39
39	IV.18	19 00 54	11 26 11.00	4 21 48	16 05.2	23 11 03.33	0 30 55
40	19	19 32 28	12 02 08.47	-2 21 27	16 16.9	0 05 43.07	1 57 00
41	20	21 59 48	15 23 33.27	-24 22 07	16 45.7	3 38 36.80	5 36 18
42	16	19 19 51	14 59 53.73	-22 10 38	16 36.3	3 15 37.73	4 55 14
43	11	21 16 22	3 55 45.40	25 05 49	15 22.1	16 30 11.13	-4 36 51
44	1952. III. 9	21 06 43	10 00 53.47	11 46 41	15 15.3	21 52 43.07	0 21 17
45	15	22 54 29	14 15 46.53	-22 05 35	15 44.8	2 35 20.07	8 00 02
46	IV. 5	22 04 23	9 46 40.40	13 58 41	15 09.7	21 36 41.80	-0 32 25
47	V. 4	19 27 57	10 55 37.07	14 43 05	15 22.5	22 38 15.47	-7 15 39
48	9	23 53 17	15 08 11.27	-23 46 52	16 04.4	3 24 42.67	5 56 13
49	VI. 3	20 11 12	12 55 03.13	-11 05 16	15 37.1	1 07 50.13	4 47 30
50	10	23 21 21	20 04 05.60	-23 52 38	16 36.2	7 52 55.87	3 05 09
51	VII. 2	19 44 23	16 13 36.00	-19 14 03	15 47.0	2 30 40.80	5 27 52
52	4	20 45 48	16 12 03.13	-27 10 54	16 16.1	4 28 13.20	5 57 01
53	1952.VIII. 4	20 ^h 33 ^m 17 ^s	20 ^h 00 ^m 32 ^s .60	-23 ^o 24'31"	16'51".0	7 ^h 49 ^m 52 ^s .67	2 ^o 47'23"
54	6	21 31 08	22 07 48.33	-11 32 33	16 55.9	9 59 23.67	0 00 51
55	IX. 8	21 19 21	3 05 15.93	22 04 05	15 54.8	15 20 13.53	-4 28 27
56	10	19 46 44	4 58 33.93	26 46 41	15 22.4	17 05 09.00	-4 03 32
57	30	19 41 32	22 08 44.80	-11 20 36	16 45.4	10 00 32.27	-0 05 28
58	X. 7	0 44 30	3 45 18.33	24 36 06	16 06.5	15 58 27.93	-4 38 13
59	XI.10	0 00 13	9 36 13.33	13 26 34	14 59.5	21 27 48.20	0 47 47
60	XII. 7	1 02 12	9 21 06.47	14 40 17	15 10.2	21 12 20.00	0 46 55
61	10	12 01 32	11 33 01.67	-1 13 09	15 06.4	23 37 10.40	3 47 49
62	1953.VII. 1	0 35 36	22 06 35.13	-9 55 50	16 33.3	10 00 32.00	-1 35 59
63	IX.19	20 32 37	20 48 01.67	-17 53 29	16 27.3	8 38 14.13	1 40 04
64	23	20 43 56	0 31 40.20	7 44 57	17 07.5	12 41 18.93	-3 58 45
65	X.27	0 41 43	6 16 27.87	25 04 02	16 11.2	18 14 55.07	-1 40 26
66	28	0 15 00	7 13 21.87	22 57 38	15 52.0	19 07 22.40	-0 34 49
67	XI.14	17 00 44	21 52 27.13	-10 56 11	16 17.9	9 46 05.45	-1 51 06
68	1954. II.18	21 39 49	10 48 38.73	2 46 42	15 13.4	22 49 57.27	4 25 55
69	24	1 57 47	15 15 17.50	-23 24 40	15 03.3	3 30 36.9	5 08 45
70	IV.12	23 42 00	9 41 50.8	9 51 00	15 08.5	21 37 41.67	3 44 43

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
71	1954. IV. 17	20 ^h 54 ^m 03 ^s	13 ^h 18 ^m 54 ^s .67	-14 ^o 06'42"	14'54".0	1 ^h 34 ^m 05 ^s .20	5 ^o 21'38"
72	Y. 11	20 20 56	10 53 29.20	1 34 01	15 08.3	22 56 17.73	5 05 27
73	I 4	19 19 16	13 04 00.60	-13 05 15	14 55.3	1 19 03.73	5 47 28
74	VI. 9	19 17 06	12 05 35.60	- 7 05 01	15 01.0	0 16 26.40	5 56 27
75	II	19 29 05	13 35 47.30	-16 19 00	14 55.2	1 52 34.67	5 53 49
76	15	21 54 00	17 04 13.53	-26 05 39	15 09.5	5 09 55.60	3 15 06
77	VII. 24	0 54 07	2 43 37.93	21 55 14	16 31.7	15 00 46.53	-5 47 01
78	VIII. 10	19 08 10	18 10 51.07	-25 21 58	15 22.8	6 09 48.60	1 56 33
79	I 2	19 15 27	20 02 13.33	-20 04 40	15 48.8	7 54 09.73	-0 23 23
80	I 4	19 10 30	21 49 27.53	-10 28 52	16 09.9	9 43 56.07	-2 31 41
81	17	21 09 36	0 30 25.33	7 57 49	16 34.0	12 30 31.13	-4 17 54
82	19	1 06 17	1 32 24.00	14 25 37	16 43.0	13 46 44.20	-4 26 18
83	21	0 22 27	3 28 51.93	22 12 52	16 30.2	15 41 31.93	-3 11 41
84	1955. Y. 4	20 38 54	13 08 57.13	-13 15 35	15 08.4	1 23 47.67	5 29 21
85	6	22 04 28	14 48 00.87	-20 36 44	14 55.7	3 03 06.87	4 13 47
86	10	1 13 16	18 03 52.13	-23 55 45	14 51.8	6 03 32.87	0 29 08
87	VI. 8	0 32 30	19 40 28.67	-20 07 12	14 57.8	7 33 59.33	-1 18 02
88	1962. IX. 13	22 55 46	23 18 31.80	- 8 48 08	17 00.0	11 08 05.13	3 59 35
89	1962. IX. 14	21 ^h 21 ^m 38 ^s	0 ^h 14 ^m 01 ^s .80	-3 ^o 55'01"	17'02".1	12 ^h 06 ^m 37 ^s .87	4 ^o 59'18"
90	15	21 51 07	1 12 24.27	1 35 48	16 58.9	13 09 13.13	5 37 27
91	18	21 47 30	4 03 24.40	15 07 36	16 20.0	16 07 14.67	5 30 37
92	X. 15	20 04 38	3 36 10.60	13 39 16	16 43.0	15 40 05.93	5 31 50
93	15	21 39 16	3 39 18.87	13 57 11	16 49.1	15 43 21.07	5 25 02
94	16	0 03 00	3 43 50.13	14 22 08	16 51.3	15 48 01.13	5 15 40
95	16	0 12 00	3 43 53.07	14 23 35	16 51.3	15 48 05.27	5 14 24
96	XII. 10	19 49 21	4 39 46.53	17 36 53	16 42.8	16 43 28.33	4 30 28
97	10	20 43 00	41 46.93	17 43 38	16 43.2	16 45 26.00	4 27 40
98	1963. I. 16	3 13 20	12 31 29.07	1 11 18	15 06.4	0 27 02.00	-4 12 54
99	II. 4	17 43 01	5 53 43.67	20 11 42	16 06.5	17 54 06.27	3 14 29
100	5	18 53 43	6 53 49.27	20 49 44	15 58.6	18 50 16.60	2 01 49
101	9	4 25 29	9 54 49.47	14 18 20	15 20.5	21 43 42.07	-1 30 55
102	12	23 24 09	12 03 30.30	3 19 14	15 09.3	0 02 31.20	3 53 34
103	13	1 29 02	12 57 37.93	-1 25 38	15 05.9	0 55 16.00	4 21 55
104	III. 4	18 50 20	6 39 23.73	20 55 02	16 03.0	18 36 48.27	2 12 47
105	4	20 22 30	6 41 44.07	20 53 06	16 00.0	18 38 59.67	2 12 25
106	5	14 58 40	7 29 01.93	20 35 57	15 47.7	19 23 05.27	1 15 15

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
107	1963.III.6	17 ^h 51 ^m 14 ^s	8 ^h 29 ^m 13 ^s .40	19°07'02"	15'42".3	20 ^h 19 ^m 44 ^s .40	-0°05'00"
108	6	19 19 20	8 31 31.67	19 00 52	15 42.2	20 21 57.20	-6 56 07
109	9	20 11 35	11 04 24.33	9 21 39	15 16.1	22 54 20.67	3 08 56
110	9	20 59 50	11 05 21.80	9 12 14	15 16.9	22 55 27.60	-3 05 44
111	10	22 13 00	11 53 34.73	4 47 08	15 11.1	23 46 29.20	3 45 06
112	IV.4	18 27 42	10 00 52.40	14 22 32	15 26.9	21 49 06.80	-2 05 00
113	5	19 25 00	10 50 42.00	10 32 48	15 16.1	22 40 02.20	-2 56 56
114	5	19 41 05	10 51 13.33	10 29 54	15 17.1	22 40 35.20	2 57 11
115	6	20 25 32	11 38 26.80	6 14 43	15 10.3	23 30 18.73	-3 35 34
116	7	20 54 33	12 24 14.33	1 47 39	15 04.4	0 19 27.80	-4 03 40
117	8	21 35 50	13 09 46.13	- 2 48 52	15 00.5	1 08 33.40	-4 17 37
118	9	20 43 29	13 53 17.60	- 6 56 22	14 54.5	1 55 09.20	-4 23 21
119	V.2	17 42 47	10 35 34.20	11 57 08	15 23.6	22 24 27.33	2 53 12
120	3	17 14 05	11 22 16.00	7 59 27	15 33.4	22 12 46.20	3 36 36
121	3	17 46 00	11 22 58.47	7 54 01	15 14.5	23 13 33.47	3 35 44
122	5	18 02 37	12 53 15.13	- 0 55 42	14 59.8	0 50 26.53	-4 24 04
123	7	20 05 47	14 25 16.27	- 9 57 31	14 56.5	2 29 05.67	-4 12 39
124	1963.V.7	20 ^h 47 ^m 10 ^s	14 ^h 26 ^m 09 ^s .27	-10°04'46"	14'56".9	2 ^h 30 ^m 04 ^s .60	-4°09'58"
125	8	23 27 54	15 16 20.47	-14 21 56	14 56.9	3 22 05.93	-3 38 53
126	9	21 49 00	16 01 41.07	-17 27 52	14 58.1	4 07 27.27	-3 08 13
127	9	22 23 34	16 02 32.87	-17 32 32	14 58.3	4 08 19.40	-3 06 04
128	VI. 2	19 13 25	13 26 28.47	- 4 17 12	15 00.2	1 26 20.87	4 27 01
129	3	20 17 45	14 12 30.73	- 8 48 25	14 59.1	2 15 39.33	-4 16 01
130	4	21 32 22	14 59 55.37	-13 01 53	14 57.8	3 05 16.13	-3 50 57
131	5	19 01 00	15 43 52.30	-16 13 52	14 57.8	3 49 43.73	-3 26 53
132	6	20 43 59	16 35 44.93	-19 25 55	15 02.8	4 40 40.33	-2 34 25
133	30	18 06 34	13 55 23.47	- 7 03 35	15 01.5	1 57 16.73	4 27 33
134	VII. 3	20 10 15	16 18 40.07	-18 31 04	15 04.0	4 24 07.67	-2 50 35
135	5	20 06 30	18 03 01.30	-22 31 04	15 15.1	6 02 48.00	-5 52 96
136	5	20 47 38	18 04 07.60	-22 33 13	15 16.4	6 03 48.73	-5 31 36
137	6	19 36 32	18 57 03.87	-22 58 37	15 21.8	6 52 31.80	10 53 05
138	7	21 08 30	19 55 42.53	-22 16 54	15 32.8	7 46 23.53	1 28 09
139	9	23 03 28	21 50 01.07	-17 10 25	15 51.3	9 35 27.53	3 49 23
140	11	1 19 39	22 47 36.80	-12 48 55	16 01.4	10 33 46.80	4 45 38
141	12	0 13 25	23 38 44.93	- 8 18 39	16 10.2	11 27 17.67	5 31 04

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
142	1963.VII.12	23 ^h 54 ^m 23 ^s	0 ^h 30 ^m 34 ^s .40	-3°14'01"	16'14".4	12 ^h 22 ^m 56 ^s .60	6°00'12"
143	13	0 59 47	0 32 24.93	-3 00 00	16 19.4	12 25 00.80	5 58 15
144	13	1 21 22	0 33 00.07	-2 56 16	16 20.1	12 25 39.13	5 58 18
145	13	1 56 00	0 33 53.40	-2 47 52	16 20.6	12 26 41.67	5 55 51
146	29	17 27 15	15 09 50.80	-13 44 39	14 59.1	3 15 21.07	-3 49 46
147	29	18 01 33	15 10 36.67	-13 49 39	14 58.8	3 16 09.47	-3 47 57
148	29	18 29 53	15 11 15.60	-13 53 38	14 58.3	3 16 50.20	-3 46 39
149	30	16 55 50	15 56 45.27	-17 07 23	15 01.5	4 02 33.67	-3 14 07
150	31	19 22 53	16 50 06.73	-20 10 11	15 07.5	4 54 28.20	-2 17 11
151	VIII. 1	18 48 00	17 41 21.87	-22 02 30	15 13.5	5 42 50.87	-1 20 02
152	1	20 15 45	17 43 47.73	-22 07 07	15 14.1	5 44 59.20	-1 16 23
153	2	18 38 21	18 35 39.60	-22 57 06	15 23.0	6 32 49.20	-0 14 21
154	2	19 16 37	18 36 30.67	-22 58 22	15 23.4	6 33 35.67	-1 22.16
155	3	19 36 30	19 33 05.60	-22 43 22	15 34.3	7 25 30.53	6 32 16
156	6	1 02 40	21 35 39.33	-17 58 10	15 57.2	9 21 33.67	3 22 07
157	6	23 15 00	17 58 34.47	-14 20 14	15 59.5	10 12 28.13	4 15 53
158	7	0 02 00	22 29 55.93	-14 13 40	16 00.2	10 15 39.93	4 27 43
159	7	0 02 30	22 28 46.60	-14 12 59	16 08.4	10 14 38.27	4 20 54
160	1963.VIII.7	22 ^h 18 ^m 56 ^s	23 ^h 20 ^m 58 ^s .47	-9°16'06"	16'09".8	11 ^h 08 ^m 31 ^s .47	5°16'18"
161	8	21 41 47	0 13 19.13	-9 56 20	16 18.7	12 04 16.73	5 50 53
162	8	22 03 14	0 13 53.00	-9 51 49	16 19.4	12 04 58.73	5 50 31
163	8	22 23 58	0 14 29	-9 47 27	16 21.1	12 05 39	5 50 07
164	8	23 04 00	0 15 38.80	-9 33 54	16 22.1	12 06 56.68	5 49 11
165	11	23 06 05	2 56 21.47	-10 57 30	16 23.0	14 59 32.40	5 35 17
166	12	2 11 56	3 02 06.00	-11 37 01	16 34.6	15 05 42.87	5 21 20
167	28	16 56 40	17 18 35.80	-21 26 33	15 07.6	5 21 28.53	-1 39 24
168	28	17 47 26	17 19 54.67	-21 30 02	15 08.4	5 22 39.87	-1 37 09
169	28	18 55 19	17 21 37.40	-21 33 41	15 07.4	5 24 19.07	-1 35 11
170	29	16 42 57	18 11 10.00	-22 46 45	15 15.1	6 10 17.73	-0 38 25
171	29	17 44 37	18 12 48.27	-22 49 18	15 16.4	6 11 48.07	-0 35 24
172	29	18 41 34	18 14 17.80	-22 50 43	15 16.1	6 13 10.53	0 33 29
173	29	20 09 13	18 16 43.13	-22 51 10	15 14.7	6 15 24.33	0 33 29
174	30	17 29 46	19 07 19.00	-23 04 16	15 27.1	7 01 47.67	0 13 29
175	30	19 53 00	19 11 15.87	-23 01 51	15 28.0	7 05 25.20	0 35 33
176	31	18 23 47	20 05 04.07	-22 06 46	15 40.5	7 55 01.27	1 44 10
177	IX. 1	19 53 54	21 04 20.67	-19 47 46	15 56.5	8 50 58.07	2 55 40

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
178	1963.IX.1	20 ^h 35 ^m 31 ^s	21 ^h 05 ^m 31 ^s .07	-19°42'56"	15'56".6	8 ^h 52 ^m 07 ^s .13	2°55'48"
179	2	18 25 31	21 58 15.07	-16 37 38	16 06.6	9 43 37.73	3 58 45
180	2	19 35 20	22 00 14.80	-16 28 17	16 07.8	9 45 38.60	3 59 52
181	3	19 51 52	22 56 31.20	-12 05 01	16 21.2	10 42 58.07	4 54 52
182	4	21 23 00	23 54 04.47	- 6 41 34	16 34.5	11 43 53.07	5 32 59
183	5	21 07 48	0 48 12.87	- 1 15 04	16 37.2	12 42 22.40	5 55 02
184	5	23 13 44	0 52 03.40	- 0 45 54	16 39.5	12 46 41.47	5 50 42
185	9	1 12 21	3 42 14.47	14 55 14	16 41.6	15 47 00.47	4 38 12
186	26	16 59 48	18 45 17.13	-23 19 16	15 15.1	6 41 32.60	0 17 02
187	28	18 05 07	20 37 25.13	-21 10 07	15 44.1	8 25 10.27	2 32 45
188	29	17 15 06	21 31 52.07	-18 26 59	15 58.0	9 17 25.73	3 36 33
189	29	17 43 05	21 32 39.93	-18 23 17	15 58.4	9 18 13.27	3 36 58
190	X. 2	20 01 24	0 22 54.07	- 3 45 48	16 46.5	12 15 01.87	5 43 43
191	4	21 36 38	2 18 02.47	7 58 59	16 59.1	14 19 44.27	5 29 52
192	8	1 07 57	5 23 15.67	20 32 53	16 41.6	17 25 34.87	2 37 21
193	9	2 23 12	6 26 39.67	21 57 59	16 28.6	18 24 43.47	1 20 06
194	30	19 56 07	0 51 44.53	- 0 44 07	16 52.2	12 46 26.80	5 47 13
195	31	18 02 23	1 44 36.47	4 36 53	16 53.2	13 43 43.60	5 47 43
196	1963.X.31	18 ^h 47 ^m 28 ^s	1 ^h 46 ^m 00 ^s .27	-4°48'04"	16'59".7	13 ^h 45 ^m 18 ^s .07	5°44'46"
197	XI. 1	21 42 37	2 49 22.00	11 01 57	17 11.2	14 53 02.00	5 01 02
198	1	23 05 22	2 51 46.40	11 19 29	17 10.9	14 55 38.60	4 54 41
199	2	18 12 45	3 42 42.80	15 04 04	16 58.4	15 47 35.20	4 31 08
200	2	18 30 55	3 43 24.20	15 08 08	16 59.2	15 48 17.93	4 29 24
201	2	18 41 25	3 43 47.20	15 10 28	16 59.7	15 48 41.73	4 28 23
202	25	16 17 27	23 29 27.27	- 9 24 53	16 03.0	11 17 03.73	5 37 14
203	27	16 33 51	1 16 04.67	1 34 32	16 43.7	13 12 36.80	5 59 29
204	27	17 08 36	1 17 03.07	1 42 54	16 44.5	13 13 43.20	5 57 16
205	29	20 44 00	3 17 42.27	13 26 43	17 11.7	15 22 25.13	4 37 22
206	30	21 54 52	4 21 47.33	17 58 30	17 14.6	16 26 41.33	3 30 10
207	30	23 01 18	4 23 54.60	18 03 02	17 14.1	16 28 47.20	3 25 43
208	XII. 4	0 48 47	7 40 56.67	22 01 58	16 43.3	19 33 07.33	-0 36 19
209	4	20 35 01	8 33 21.13	20 25 45	16 14.8	20 22 13.67	-1 35 34
210	24	15 02 35	0 54 33.87	- 0 46 09	16 19.5	12 49 00.47	6 05 35
211	24	15 24 20	0 55 07.07	- 0 41 13	16 19.9	12 49 38.93	6 04 15
212	24	15 48 05	0 55 45.67	- 0 35 48	16 22.3	12 50 23.20	6 03 00
213	25	14 02 00	1 45 21.20	4 29 22	16 26.7	13 44 14.60	5 58 45

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
214	1963.XII.25	14 ^h 58 ^m 21 ^s	1 ^h 46 ^m 59 ^s .60	4 ^o 42'45"	16'28.6	13 ^h 46 ^m 06 ^s .00	5 ^o 55'01"
215	25	15 22 26	1 47 40.93	4 48 26	16 29.3	13 46 52.86	5 53 23
216	26	15 20 10	2 42 27.00	10 07 22	16 38.0	14 45 25.53	5 22 36
217	26	16 13 52	2 44 06.93	10 19 48	16 45.6	14 47 14.73	5 18 11
218	29	17 42 51	5 50 13.46	21 34 37	17 00.4	17 50 54.33	1 50 53
219	29	18 43 53	5 52 24.46	21 41 10	17 02.2	17 52 56.53	1 44 48
220	1964.I. 23	15 31 00	3 20 57.53	13 36 08	16 37.8	15 25 38.93	4 40 29
221	25	16 37 42	5 22 07.00	20 45 53	16 46.2	17 24 34.00	2 23 20
222	26	20 51 44	6 33 39.93	22 26 38	16 49.4	18 31 06.13	0 46 24
223	28	18 48 16	8 35 13.60	20 27 19	16 30.2	20 23 54.20	-1 43 57
224	28	18 59 15	8 35 36.07	20 26 44	16 30.6	20 24 15.20	-1 44 22
225	29	19 14 29	9 34 58.20	17 24 02	16 16.4	21 21 34.07	-2 51 23
226	II. 21	16 40 18	5 04 30.60	20 19 13	16 36.8	17 07 58.07	2 29 59
227	21	17 24 58	5 05 51.73	20 23 59	16 36.8	17 09 16.13	2 27 02
228	23	16 32 40	7 07 13.33	22 32 53	16 31.1	19 01 56.93	-0 10 04
229	23	17 33 52	7 09 12.07	22 33 45	16 31.4	19 03 45.40	-4 13 09
230	24	17 57 10	8 09 09.80	21 25 55	16 23.9	19 59 18.80	-1 16 11
231	24	18 16 10	8 11 04.87	21 23 26	16 24.4	20 01 05.53	1 19 29
232	1964.II.24	16 ^h 57 ^m 40 ^s	8 ^h 11 ^m 41 ^s .00	21 ^o 22'29"	16'24.7	20 ^h 01 ^m 39 ^s .27	1 ^o 20'23"
233	II.20	17 16 50	9 34 58.40	17 33 39	16 06.6	21 21 08.80	-3 10 07
234	20	17 40 50	9 35 38.20	17 33 20	16 06.6	21 21 49.20	-3 09 52
235	21	19 23 06	10 31 12.27	13 25 01	15 53.4	22 18 00.93	-3 49 20
236	22	18 01 42	11 19 15.20	9 36 06	15 43.1	23 08 16.40	-4 20 19
237	22	18 32 42	11 19 58.87	9 36 07	15 43.4	23 09 05.60	-4 19 03
238	23	20 00 06	12 09 44.93	3 37 13	15 31.4	0 02 54.53	-4 26 36
239	24	21 43 50	12 58 20.40	-1 34 27	15 22.9	0 56 12.07	-4 16 02
240	26	20 32 36	14 28 32.60	-10 56 54	15 04.2	2 33 25.13	-3 31 49
241	26	21 07 50	14 29 19.37	-11 03 48	15 04.6	2 34 18.00	-3 28 57
242	26	22 56 15	14 31 41.87	-11 23 57	15 04.7	2 36 55.87	-3 20 49
243	27	21 49 00	15 17 00.00	-15 22 11	14 59.6	3 23 45.80	-2 43 12
244	28	23 18 00	16 07 03.53	-19 07 59	14 55.0	4 13 45.07	-1 44 42
245	28	23 46 00	9 27 43.47	-15 11 42	14 55.0	4 14 25.00	-1 42 50
246	30	1 19 12	16 59 34.57	-22 03 34	14 52.1	5 04 05.47	-0 39 18
247	Y.23	20 16 00	14 14 55.47	-9 42 33	15 04.7	2 19 06.60	-3 36 52
248	29	23 08 10	19 16 22.73	-24 33 53	14 55.6	7 09 17.47	2 15 41
249	31	0 10 36	20 11 06.87	-23 26 38	15 00.5	7 59 21.00	3 19 53

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
250	1964.VI.18	21 ^h 42 ^m 00 ^s	13 ^h 20 ^m 20 ^s .27	- 3 ^o 58' 44"	15' 09".6	1 ^h 20 ^m 11 ^s .53	-4 ^o 09' 57"
251	19	16 24 00	13 58 29.13	- 7 42 12	15 10.8	2 01 09.20	-4 03 40
252	19	16 50 30	13 59 04.07	- 7 51 37	15 11.2	2 01 49.33	-4 01 35
253	19	17 07 30	13 59 26.20	- 7 55 05	15 10.4	2 02 14.73	-4 00 13
254	19	17 27 30	13 59 51.13	- 7 59 06	15 10.5	2 02 48.53	-3 58 36
255	19	18 15 30	14 00 52.27	- 8 08 36	15 10.6	2 03 53.60	-3 54 54
256	19	18 34 00	14 01 15.87	- 8 12 11	15 10.6	2 04 20.53	-3 53 33
257	20	18 05 30	14 46 22.47	-12 36 41	15 02.4	2 52 06.40	-3 17 36
258	21	15 46 30	15 29 37.53	-16 11 02	14 50.9	3 36 21.33	-2 41 51
259	21	16 33 00	15 30 49.67	-16 19 48	14 52.2	3 37 37.07	-2 37 32
260	21	16 54 30	15 31 21.73	-16 23 43	14 52.7	3 38 10.80	-2 35 35
261	21	17 26 30	15 32 08.80	-16 29 26	14 55.3	3 39 00.07	-2 32 45
262	21	18 12 00	15 33 12.40	-16 37 18	14 56.1	3 40 06.87	-2 28 45
263	22	17 26 00	16 20 11.53	-19 49 51	14 49.6	4 26 26.27	-1 36 40
264	22	18 36 00	16 21 57.07	-20 00 29	14 52.0	4 28 11.20	-1 30 19
265	22	18 50 30	16 22 18.13	-20 02 33	14 52.2	4 28 32.13	-1 25 06
266	22	19 15 30	16 22 54.07	-20 06 02	14 53.5	4 29 07.67	-1 27 02
267	22	20 23 30	16 24 29.87	-20 14 44	14 53.7	4 30 42.07	-1 22 07
268	1964.VI.23	20 ^h 19 ^m 00 ^s	17 ^h 14 ^m 24 ^s .00	-22 ^o 41' 49"	14' 52".0	5 ^h 17 ^m 58 ^s .33	-0 ^o 20' 02"
269	23	22 28 00	17 17 26.53	-22 51 36	14 51.7	5 20 49.13	-0 13 29
270	24	20 15 00	18 05 47.87	-24 11 33	14 51.1	6 05 17.33	0 45 12
271	25	19 44 00	18 57 23.13	-24 37 03	14 49.4	6 52 06.00	1 49 05
272	25	20 59 00	18 59 27.33	-24 39 08	14 50.9	6 53 57.33	1 54 00
273	25	20 22 00	18 57 12.73	-24 40 47	14 51.7	6 51 55.07	1 52 34
274	26	20 47 00	19 51 52.33	-23 57 44	14 54.9	7 41 38.93	2 57 02
275	26	21 48 30	19 53 33.13	-23 56 58	14 56.1	7 43 10.40	3 00 18
276	26	22 25 00	19 54 30.53	-23 55 31	14 56.6	7 44 02.80	3 01 41
277	29	22 16 00	22 27 51.80	-15 33 47	15 16.3	10 11 49.47	5 31 07
278	30	0 05 00	22 30 44.40	-15 18 05	15 24.2	10 14 48.13	5 31 52
279	VII.1	0 54 00	23 21 08.60	-10 39 57	15 37.4	11 07 31.13	5 57 34
280	2	0 58 00	0 09 56.60	- 5 36 23	15 50.3	12 00 10.13	6 07 52
281	18	18 50 00	15 20 15.20	-15 44 46	14 59.4	3 27 10.93	-2 33 31
282	19	18 37 20	16 07 54.33	-19 14 36	14 55.0	4 14 37.20	-1 40 27
283	20	19 25 30	16 58 23.47	-22 04 05	14 52.3	5 03 00.00	-0 37 03
284	21	20 09 34	17 50 35.87	-23 54 58	14 52.9	5 11 24.33	0 29 21
285	21	20 53 20	17 51 43.53	-23 56 52	14 52.6	5 52 26.27	0 31 00

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
286	1964.VII.22	20 ^h 32 ^m 24 ^s	18 ^h 43 ^m 36 ^s .87	-24°40'02"	14'52.7	6 ^h 39 ^m 36 ^s .53	1°35'49"
287	22	21 18 43	18 44 47.80	-24 40 11	14 53.6	6 40 40.73	1 37 11
288	23	20 19 00	19 36 19.67	-24 17 39	14 59.5	7 27 25.93	2 39 32
289	24	23 39 25	20 34 22.67	-22 31 51	15 05.9	8 21 04.07	3 41 17
290	27	0 12 00	22 17 59.33	-16 11 50	15 24.0	10 02 00.73	5 14 42
291	VIII. 18	18 45 00	18 23 52.33	-24 37 14	14 55.7	6 21 42.07	1 17 19
292	21	19 43 25	21 04 15.13	-21 12 34	15 14.2	8 49 15.47	4 16 31
293	26	23 14 30	1 20 52.27	2 33 39	16 06.9	13 18 33.20	5 31 48
294	28	23 19 30	3 04 59.80	13 03 20	16 18.7	15 10 02.87	4 10 12
295	29	0 27 00	3 06 58.20	13 18 20	16 23.8	15 12 10.53	4 03 42
296	IX.15	16 28 10	18 54 33.33	-24 58 14	14 57.5	6 49 24.33	2 06 33
297	15	17 10 10	18 55 37.87	-24 57 53	14 57.7	6 50 22.73	2 07 35
298	16	18 51 22	19 51 11.00	-24 11 52	15 03.8	7 40 51.27	3 09 15
299	17	18 46 52	20 43 51.60	-22 20 14	15 11.4	8 29 45.60	4 03 35
300	20	19 20 35	23 18 40.27	-10 43 44	15 47.4	11 05.10.53	5 46 42
301	23	21 23 50	1 53 54.13	6 17 40	16 19.2	13 54 48.73	5 02 47
302	26	22 17 00	4 43 02.33	20 14 52	16 29.6	16 47 55.00	2 00 03
303	X.14	16 20 15	20 20 28.00	-23 33 22	15 00.0	8 07 37.73	3 55 06
304	1964.X.15	16 ^h 22 ^m 00 ^s	21 ^h 12 ^m 22 ^s .47	-21°05'55"	15'08.3	8 ^h 56 ^m 39 ^s .33	4°43'22"
305	15	16 39 00	21 12 48.00	-21 04 06	15 08.5	8 57 04.27	4 43 24
306	16	16 27 10	22 03 33.20	-17 36 09	15 19.6	9 46 59.67	5 21 47
307	21	20 00 00	2 24 57.00	9 36 55	16 26.0	14 28 21.00	4 30 42
308	24	19 48 00	5 21 20.00	22 35 07	16 27.4	17 24 11.33	1 03 41
309	26	0 44 00	6 35 35.75	23 57 40	16 30.6	18 32 30.60	-4 55 50
310	XI.14	21 56 00	29 30 33.93	-9 24 56	15 33.7	11 18 04.40	5 43 42
311	16	18 53 00	1 05 58.27	1 01 11	16 09.7	13 02 21.60	5 32 40
312	18	17 00 00	2 49 58.20	11 57 29	16 32.3	14 54 41.67	5 10 41
313	21	28 28 00	5 58 26.53	23 16 27	16 41.9	17 58 34.20	0 08 20
314	21	19 02 00	5 59 53.47	23 22 42	16 43.4	17 59 54.00	0 04 06
315	21	21 15 00	6 05 06.40	23 37 33	16 47.3	18 04 40.73	-0 11 04
316	21	21 50 00	6 06 21.47	23 40 49	16 48.2	18 05 49.33	-0 14 30
317	1965.I. 12	16 02 30	2 54 04.60	13 35 12	16 14.5	14 59 16.67	3 52 10
318	13	17 15 00	3 52 28.40	17 31 58	16 32.1	15 58 53.87	2 37 30
319	14	15 50 00	4 50 28.67	20 59 14	16 40.5	16 55 12.00	1 29 15
320	II. 10	15 34 00	4 27 16.93	20 01 39	16 21.8	16 33 08.67	1 41 18

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
321	1965.II.10	15 ^h 50 ^m 00 ^s	4 ^h 27 ^m 55 ^s .47	20°05'05"	16'22".1	16 ^h 33 ^m 46 ^s .60	1°39'19"
322	12	15 48 00	6 33 07.80	24 07 51	16 39.3	18 30 13.27	-0 54 05
323	12	16 24 00	6 34 27.67	24 10 11	16 40.4	18 31 25.47	-0 57 28
324	12	18 03 00	6 37 52.27	24 14 50	16 43.4	18 34 30.60	-1 05 01
325	14	18 06 00	8 46.45.73	21 14 56	16 43.3	20 35 18.73	-3 18 35
326	14	19 22 00	8 51 22.87	21 07 34	16 48.3	20 37 48.00	-3 21 21
327	III.15	19 25 00	10 26 54.00	14 25 36	16 29.1	22 12 37.87	-4 22 34
328	15	18 50 00	11 21 06.27	9 08 40	16 19.8	23 09 53.47	-4 33 25
329	16	19 44 00	11 22 38.07	8 56 44	16 21.3	23 11 35.87	-4 31 22
330	24	8 52 00	17 30 55.27	-24 44 33	14 56.7	5 33 35.67	1 27 45
331	IV.10	16 55 00	9 06 31.73	20 37 49	16 22.4	20 51 58.20	-3 52 21
332	10	17 18 00	9 07 12.20	20 34 43	16 22.6	20 52 38.13	-3 52 09
333	11	19 15 00	10 08 21.33	16 04 40	16 18.8	21 53 27.60	-4 20 18
334	12	17 28 00	11 00 12.27	11 24 21	16 12.4	22 47 21.33	-4 38 07
335	13	18 46 00	11 54 31.20	5 29 02	16 11.7	23 46 13.87	-4 29 08
336	15	19 59 00	13 36 08.87	-6 25 38	15 45.3	1 38 28.80	-3 20 44
337	15	20 31 00	13 36 56.93	-6 33 23	15 45.9	1 39 24.60	-3 17 51
338	15	21 48 00	13 38 45.93	-6 51 41	15 46.6	1 41 32.27	-3 10 37
339	1965.V.11	18 ^h 25 ^m 00 ^s	12 ^h 29 ^m 35 ^s .20	1°25'49"	15'52".3	0 ^h 24 ^m 54 ^s .0	-4°15'00"
340	12	17 20 00	13 16 38.37	-4 03 14	15 41.0	1 16 53.13	-3 45 04
341	12	19 44 00	13 20 04.00	-4 37 47	15 42.0	1 20 55.07	-3 32 15
342	14	19 52 40	14 57 57.13	-15 06 13	15 22.5	3 05 46.53	-1 43 40
343	16	20 15 00	16 40 28.80	-22 42 41	15 04.1	4 46 52.27	0 31 05
344	VI.11	19 11 00	15 32 20.53	-18 09 26	15 12.2	3 40 46.73	-0 56 21
345	15	21 57 00	19 06 16.73	-26 10 55	14 48.7	6 59 25.93	3 35 10
346	15	22 46 30	19 07 37.53	-26 10 56	14 49.2	7 00 38.07	3 37 16
347	16	22 16 00	19 59 28.13	-25 10 19	14 46.4	7 47 30.47	4 28 12
348	16	22 42 30	20 00 10.13	-25 09 23	14 46.8	7 48 08.60	4 29 10
349	16	23 17 00	20 01 03.33	-25 07 53	14 47.2	7 48 57.07	4 30 07
350	VII. 6	19 35 00	13 43 04.00	-7 31 44	15 34.7	1 46 29.27	-2 56 20
351	7	19 37 00	14 31 14.27	-12 43 53	15 23.5	2 38 10.67	-2 03 04
352	8	19 30 00	15 19 51.33	-17 17 05	15 13.2	3 28 23.33	-1 02 54
353	10	18 18 00	16 59 39.73	-23 46 10	14 57.4	5 04 52.67	1 02 31
354	10	18 42 00	16 58 51.93	-23 49 00	14 46.6	5 04 10.27	1 06 28
355	11	19 00 00	17 53 23.27	-25 36 41	14 45.8	5 54 02.00	2 10 23
356	12	23 42 00	18 54 02.60	-26 12 00	15 03.2	6 48 29.07	3 19 12

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TABLE 1, Continued

No.	Date	UT	α'	δ'	h'	λ'	β'	/158
357	1965.VII.13	21 ^h 20 ^m 00 ^s	19 ^h 43 ^m 12 ^s .07	-25°34'44"	15'23.4	7 ^h 32 ^m 43 ^s .33	4°11'07"	
358	13	22 10 30	19 44 30.47	-25 33 34	15 35.7	7 33 54.67	4 12 04	
359	20	0 27 00	0 33 49.67	- 2 20 30	15 05.3	12 27 21.67	5 30 22	
360	VII. 6	16 16 30	16 42 42.80	-23 04 55	14 47.6	4 49 06.47	0 49 00	
361	6	17 51 00	16 45 04.93	-23 16 19	14 51.1	5 51 22.00	0 56 04	
362	7	18 03 00	17 37 46.40	-25 21 20	14 54.5	5 39 54.47	2 00 21	
363	8	19 41 00	18 33 22.27	-26 17 56	14 58.3	6 29 56.40	3 04 07	
364	9	18 41 00	19 24 48.40	-25 59 08	15 03.8	7 16 04.20	3 55 42	
365	10	18 49 00	20 17 07.07	-24 31 57	14 56.4	8 03 45.13	4 41 37	
366	10	20 08 00	20 19 08.27	-24 27 16	14 52.5	8 05 37.13	4 43 17	
367	15	21 34 00	0 17 47.07	- 4 04 53	14 50.7	12 09 48.93	5 30 46	
368	18	1 28 00	1 54 47.47	7 32 55	14 53.6	13 57 24.27	3 57 01	
369	19	1 41 30	2 43 57.33	12 44 55	14 54.3	14 50 01.00	2 59 11	
370	IX. 3	16 52 30	17 20 11.73	-25 02 48	15 10.7	5 23 57.07	1 54 40	
371	4	16 47 00	18 13 24.53	-26 19 15	15 25.4	6 12 01.93	2 54 33	
372	6	18 34 00	20 01 56.80	-25 12 47	15 01.6	7 49 40.67	4 37 21	
373	8	20 11 00	21 44 40.67	-19 31 07	15 02.9	9 27 31.13	5 36 31	
374	9	22 30 00	22 35 23.27	-15 01 12	14 56.6	10 19 24.40	5 41 10	
375	1965.IX.10	19 ^h 49 ^m 00 ^s	23 ^h 17 ^m 59 ^s .78	-10°48'54"	14'51.5	11 ^h 04 ^m 25 ^s .47	5°47'30"	
376	11	19 05 00	0 02 17.13	- 5 49 18	14 47.3	11 52 48.07	5 34 00	
377	11	20 31 00	0 04 20.13	- 5 31 10	15 08.8	11 55 09.93	5 29 37	
378	12	20 52 00	0 50 07.80	- 5 01 22	15 16.5	12 46 04.27	4 58 27	
379	16	21 01 30	4 07 21.67	19 53 22	15 53.3	16 14 36.07	1 01 03	
380	17	1 22 45	4 16 11.33	20 37 18	15 58.5	16 23 16.00	0 40 18	
381	X. 3	16 36 00	19 42 10.47	-26 01 11	14 51.4	7 31 30.13	4 34 46	
382	4	18 23 00	20 36 35.27	-23 57 51	14 49.8	8 21 35.93	5 12 03	
383	6	17 56 00	22 13 23.53	-17 16 35	14 55.5	9 56 19.67	5 51 22	
384	10	21 14 00	1 22 50.33	4 06 06	15 30.3	13 22 41.80	4 17 08	
385	11	0 28 30	1 28 19.93	4 51 33	15 31.7	13 28 55.40	4 05 27	
386	XI. 2	16 02 00	2 58 29.33	-19 05 36	14 52.7	14 21 47.33	4 20 52	
387	3	16 42 00	22 42 18.87	-14 44 14	14 58.7	10 26 04.33	6 03 07	
388	4	18 51 00	23 30 50.27	- 9 26 32	15 07.5	11 18 16.73	5 46 45	
389	9	23 00 00	3 39 54.27	18 21 16	16 05.8	15 47 56.80	1 10 02	
390	11	0 49 00	4 40 50.93	22 26 39	16 14.0	16 47 03.87	-0 14 30	
391	XII. 7	2 56 30	3 25 51.27	17 14 46	16 07.5	15 33 53.47	1 26 55	
392	1	18 19 00	4 03 15.40	20 03 57	16 16.4	16 10 56.93	0 39 28	

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TABLE 1, Continued

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No.	Date	UT	α'	δ'	h'	λ'	β'
393	1965.XII.9	20 ^h 57 ^m 00 ^s	6 ^h 03 ^m 57 ^s .80	25°27'26"	16'34.2	18 ^h 10 ^m 48 ^s .47	-2°02'30"
394	9	21 08 00	6 16 24.80	25 22 00	16 31.7	18 14 50.20	-1 58 32
395	1966.II. 3	16 45 30	7 11 11.47	25 23 14	16 43.3	19 04 12.40	-2 55 47
396	4	18 32 00	8 25 32.93	23 20 59	16 56.2	20 12 22.00	-3 59 23
397	10	5 10 00	13 35 28.60	- 7 23 06	16 10.9	1 39 15.33	-2 23 30
398	IV.4	20 00 45	12 29 31.93	0 33 16	16 37.7	0 26 14.40	-3 26 23
399	V. 3	18 34 00	13 50 15.93	-9 28 13	16 18.3	1 55 54.80	-1 45 24
400	4	19 05 00	14 41 00.13	-15 26 22	16 06.9	2 50 32.33	-0 12 11
401	4	20 20 00	14 46 58.93	-15 44 49	16 08.3	2 56 24.13	-0 20 31
402	6	21 55 00	16 42 00.20	-24 34 26	15 42.3	4 49 15.27	2 19 10
403	VII.4	22 22 00	20 45 44.47	-24 00 07	14 58.2	8 29 40.60	5 46 49
404	5	23 46 00	20 47 12.67	-23 55 14	14 58.1	8 31 03.93	5 47 27
405	27	0 23 30	16 38 35.40	-24 21 36	15 29.9	4 46 12.73	2 29 26
406	28	18 57 00	17 35 18.20	-26 53 07	15 20.4	5 37 56.33	3 33 35
407	31	20 39 30	20 22 49.73	-25 06 05	14 58.6	8 08 17.67	5 32 48
408	VIII.1	20 49 00	21 13 57.00	-22 03 33	14 54.2	8 56 54.47	5 44 58
409	2	23 14 00	22 06 57.47	-17 43 52	14 51.3	9 49 54.80	5 44 17
410	24	19 02 20	17 21 52.40	-26 40 28	15 24.2	5 25 53.93	3 30 44

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411	1966.VIII.30	21 ^h 50 ^m 45 ^s	22 ^h 38 ^m 41 ^s .27	-14°35'21"	14'48.9	10 ^h 23 ^m 01 ^s .73	5°35'10"
412	IX. 1	0 51 00	23 27 12.33	- 8 53 24	14 49.0	11 15 43.57	4 59 40
413	5	22 06 40	3 00 31.87	16 33 34	15 16.0	15 09 48.00	0 30 15
414	23	21 36 00	19 52 59.03	-25 45 16	15 01.1	7 46 05.60	5 38 30
415	26	18 24 00	22 19 21.80	-15 28 17	14 48.4	10 02 51.00	5 37 12
416	28	18 53 00	23 46 41.07	- 6 16 06	14 40.5	11 37 48.40	4 25 41
417	X.31	20 21 00	4 15 30.93	22 31 29	15 25.3	16 23 59.40	-1 13 53
418	XI.23	16 00 00	0 45 05.47	0 41 41	14 57.0	12 42 32.53	3 49 16
419	25	20 19 00	2 20 48.47	12 28 11	15 11.5	14 28 11.87	1 28 50
420	27	18 59 00	3 57 35.27	21 13 55	15 26.6	16 06 40.40	-0 44 53
421	28	20 17 00	4 57 59.13	24 37 28	15 37.5	17 01 00.13	-2 00 32

TABLE 2

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No.	π	h	m	n	g	g'	ω	ω'
1	56' 50"	15' 41"	136° 31' 10"	6° 02' 09"	274° 53' 28"	111° 06' 32"	215° 35' 31"	276° 03' 00"
2	57 58	16 02	141 09 25	7 27 50	282 32 13	84 31 37	211 09 22	274 37 19
3	58 41	16 12	162 55 20	5 55 47	301 04 09	113 05 02	215 55 12	276 09 22
4	58 43	16 11	163 22 48	5 55 40	301 31 12	113 07 01	215 55 34	276 09 32
5	58 44	16 12	164 14 31	5 55 28	302 22 41	113 10 55	215 56 24	276 09 47
6	61 05	16 48	203 27 08	5 46 01	341 15 01	116 07 01	216 25 37	276 19 05
7	57 37	15 52	159 30 16	4 29 48	294 38 13	139 45 25	220 22 12	276 35 24
8	54 15	16 20	185 13 57	4 23 36	320 08 49	141 52 52	220 41 31	277 41 38
9	59 17	16 19	185 39 21	4 23 29	320 34 05	141 42 47	220 41 46	277 41 42
10	60 42	16 40	212 29 18	4 17 01	347 22 05	143 42 18	221 02 10	277 55 05
11	60 04	16 25	274 44 19	2 35 12	45 51 14	175 18 14	226 17 58	279 30 07
12	57 47	15 50	315 15 56	2 25 26	86 02 10	178 20 10	226 48 222	279 39

13	56' 46"	15' 35"	328° 22' 37"	2° 22' 16"	99° 02' 20"	179° 19' 01"	226° 58' 01"	279° 43' 12"
14	56 45	15 35	328 26 26	2 22 14	99 03 02	179 19 41	226 58 12	279 42 50
15	55 24	15 11	354 55 37	2 15 52	125 21 50	181 18 11	227 18 00	279 49 30
16	59 34	16 17	229 46 35	1 19 14	353 17 46	198 52 08	230 09 36	280 46 08
17	59 17	16 13	283 16 01	1 06 20	51 15 53	202 52 12	230 53 42	280 53 06
18	57 26	15 45	323 43 12	0 56 34	31 27 40	205 54 07	231 16 00	281 08 58
19	56 41	15 34	337 19 38	0 53 18	134 52 12	206 54 50	231 34 08	281 17 42
20	54 16	14 56	57 16 30	0 34 14	134 33 29	212 53 42	232 33 47	281 19 12
21	57 57	14 51	304 20 44	359 34 27	69 07 24	231 22 30	235 21 00	282 31 01
22	56 51	15 35	331 28 43	359 27 54	95 01 35	233 24 14	235 59 10	282 43 37
23	56 19	15 26	344 13 10	359 24 50	108 39 13	234 21 25	236 08 46	282 52 48
24	55 40	15 28	346 43 59	359 24 14	111 09 30	234 32 46	236 10 37	282 53 17
25	55 09	15 11	372 11 31	359 13 05	136 23 59	236 26 56	236 29 31	282 47 28
26	57 09	15 40	314 07 05	358 05 17	75 46 12	259 02 02	240 15 36	284 00 22
27	55 10	15 12	356 45 01	357 55 00	113 02 10	262 13 26	240 48 00	284 10 34
28	54 25	15 01	36 00 47	357 45 32	156 58 19	265 09 36	241 16 55	284 20 06

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
29	54' 23"	15' 01"	88° 18' 08"	357° 32' 56"	208° 49' 01"	269° 04' 19"	241° 56' 06"	284° 32' 35"
20	54 47	15 08	102 09 29	357 29 35	222 33 43	270 06 29	242 06 11	284 36 00
31	55 04	15 10	2 31 26	356 26 48	120 43 23	289 34 59	245 21 14	285 38 53
32	54 41	15 07	III 05 47	356 00 37	228 22 34	297 36 01	246 42 36	286 11 10
33	55 17	15 17	125 30 26	355 38 05	242 39 58	298 46 59	247 12 22	286 27 04
34	57 13	15 44	180 59 55	352 50 10	291 30 18	356 47 20	256 33 29	289 15 43
35	57 00	15 42	176 20 09	351 24 28	283 56 02	23 22 12	260 59 38	290 41 34
36	54 13	14 54	78 54 41	350 21 09	184 17 17	43 00 30	264 16 15	291 44 49
37	56 16	15 27	148 12 55	347 10 50	246 55 08	102 03 00	274 06 57	294 55 20
38	56 50	15 41	161 30 29	347 07 33	260 06 00	103 02 30	274 16 52	294 58 34
39	57 40	15 54	174 04 55	347 04 36	272 34 01	103 59 06	274 26 17	295 01 34
40	58 32	16 07	187 32 46	347 01 21	285 52 08	104 59 31	274 38 42	295 04 43
41	60 56	16 41	202 04 16	346 57 51	300 19 12	106 04 42	274 47 13	295 02 46
42	60 17	16 31	231 39 44	343 57 06	323 34 26	162 08 53	284 08 13	298 09 14
43	54 47	15 09	58 07 38	334 31 18	130 07 08	337 40 00	313 29 13	307 35 35
44	54 38	15 04	150 44 16	329 48 32	212 53 46	65 22 37	328 01 58	312 13 32
45	57' 23"	15' 41"	230° 46' 58"	329° 29' 15"	292° 15' 50"	71° 21' 47"	329° 01' 52"	312° 37' 56"
46	54 36	15 01	147 01 44	328 22 38	206 10 39	92 01 37	332 28 35	313 44 33
47	55 09	15 11	167 42 44	326 50 50	223 38 20	120 30 07	337 13 34	315 16 28
48	58 35	16 01	236 01 20	326 34 22	291 22 16	125 36 43	338 04 42	315 32 55
49	56 29	15 31	203 16 00	325 15 26	235 58 52	150 06 00	342 09 43	316 51 56
50	60 26	16 32	297 22 29	324 52 46	349 09 40	157 07 48	343 20 03	317 14 34
51	57 26	15 43	225 16 13	323 43 20	274 37 23	178 39 50	346 55 30	318 24 06
52	59 20	16 13	252 11 06	323 36 51	301 18 36	180 40 37	347 15 39	318 30 36
53	61 14	16 46	300 32 20	321 58 23	346 12 43	211 13 19	352 21 14	320 09 08
54	61 13	16 48	327 25 16	321 52 30	12 52 01	213 13 59	352 41 21	320 15 38
55	57 27	15 47	42 08 03	320 07 05	93 54 14	245 45 00	358 06 44	322 00 00
56	55 45	15 17	67 38 24	320 00 55	109 11 38	247 39 29	358 25 49	322 06 40
57	60 28	16 37	331 07 11	318 57 24	10 14 46	267 21 58	1 41 49	323 40 16
58	57 22	15 52	52 57 02	318 37 40	91 35 06	273 29 13	2 44 16	323 30 03
59	54 15	14 53	140 32 28	316 49 44	175 23 35	306 58 01	8 19 19	325 18 05
60	54 19	14 59	136 52 21	315 23 49	168 42 40	333 37 16	12 45 53	326 44 38

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
61	54' 33"	14' 59"	176° 56' 41"	315° 14' 09"	200° 26' 38"	336° 37' 05"	13° 15' 54"	326° 53' 45"
62	52 49	16 25	330 50 02	304 53 22	339 51 25	176 38 10	46 37 15	337 39 00
63	59 47	16 22	316 01 54	300 12 33	315 55 05	255 18 11	59 54 17	340 56 11
64	61 23	16 55	8 50 28	299 59 48	7 16 52	260 15 07	61 33 48	342 09 00
65	57 40	15 57	85 50 15	298 14 26	41 34 55	292 56 28	66 00 54	343 54 22
66	56 45	15 40	98 16 10	298 11 19	94 24 18	293 54 29	66 10 34	343 57 31
67	58 50	16 10	331 58 17	297 15 05	325 38 02	311 21 11	69 05 10	344 05 42
68	54 45	15 04	159 27 32	292 09 27	142 24 00	46 09 43	84 54 00	349 23 31
69	54 18	14 51	359 27 56	291 59 00	202 57 36	77 52 12	103 22 12	352 43 12
70	55 07	15 05	138 55 33	289 20 48	115 57 36	98 29 02	93 37 12	351 51 36
71	53 57	14 48	203 16 16	289 05 17	179 45 36	103 05 46	94 25 12	352 26 24
72	54 40	15 01	159 12 38	287 49 06	133 01 12	126 55 48	98 57 36	354 19 48
73	54 00	14 49	198 10 01	287 39 42	171 39 36	129 50 38	98 51 00	354 30 00
74	54 23	14 55	180 43 53	286 21 06	151 19 12	155 27 36	103 07 48	355 52 48
75	54 08	14 50	207 11 44	286 10 43	177 36 36	157 26 24	103 27 36	355 58 48
76	55 11	15 06	261 17 37	285 57 41	231 08 24	161 28 52	104 07 48	356 12 00
77	59 20	16 21	43 34 42	283 56 33	9 14 24	199 03 50	110 16 00	358 13 12
78	56' 00"	15' 19"	277° 35' 16"	283° 00' 06"	241° 22' 12"	266° 33' 58"	113° 19' 12"	359° 07' 48"
79	57 26	15 44	304 00 29	282 53 46	217 23 12	213 32 31	113 38 24	359 15 36
80	58 44	16 05	330 18 55	282 47 25	213 33 36	220 30 32	113 58 12	359 22 12
81	59 42	16 25	10 56 03	282 37 37	213 43 48	223 33 07	114 28 12	359 31 48
82	59 40	16 29	26 15 57	282 33 55	213 03 00	224 42 00	114 39 36	359 35 24
83	59 13	16 20	52 13 41	282 27 43	213 46 48	276 38 28	114 59 24	359 42 00
84	54 45	15 02	196 30 57	263 51 36	130 06 43	119 46 59	157 32 38	13 18 46
85	54 12	14 51	223 33 08	263 45 04	157 19 12	121 48 43	157 33 14	13 25 23
86	54 04	14 48	264 54 32	263 35 07	138 09 00	124 53 56	158 10 26	13 35 17
87	54 21	14 53	286 26 00	267 03 04	216 45 00	153 27 11	162 35 46	15 07 26
88	61 17	16 42	349 05 28	126 27 44	343 47 00	250 09 23	239 10 44	155 50 12
89	61 22	16 43	1 24 22	126 24 46	355 39 39	251 04 39	239 19 57	155 53 10
90	61 07	16 39	14 51 08	126 21 31	8 57 36	252 05 00	239 30 01	155 56 25
91	59 00	16 04	54 20 54	126 12 00	43 09 20	255 02 15	239 59 34	156 05 56
92	60 32	16 34	49 10 11	124 46 27	39 53 38	281 34 42	244 45 06	157 31 33
93	60 13	16 37	50 02 09	124 46 14	40 50 08	281 38 35	244 25 47	157 31 47
94	60 09	16 37	51 23 03	124 45 55	42 03 22	281 44 30	244 26 46	157 32 06
95	60 08	16 37	51 26 00	124 45 54	42 13 17	281 44 52	244 26 49	157 32 07
96	59 36	16 29	66 54 29	121 48 33	51 28 40	336 45 41	253 37 16	160 29 38

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
97	59' 35"	16' 29"	67° 23' 57"	121° 48' 26"	51° 57' 53"	336° 47' 54"	258° 37' 38"	160° 29' 45"
98	54 21	14 57	185 19 20	119 53 11	165 50 50	12 33 49	259 35 19	162 25 06
99	57 48	15 53	83 37 49	118 50 54	61 58 16	31 52 07	262 48 39	163 27 27
100	57 19	15 45	97 27 13	118 47 34	75 40 40	32 54 09	262 58 59	163 30 46
101	55 39	15 17	142 12 53	118 36 47	120 03 37	36 15 03	263 32 29	163 41 33
102	54 32	15 00	178 59 12	118 27 54	156 31 17	39 00 05	264 00 12	163 50 28
103	54 15	14 57	193 18 21	118 24 27	170 43 10	40 04 21	264 10 44	163 53 55
104	57 14	15 49	93 11 08	117 21 47	68 24 07	59 30 41	267 25 14	164 56 38
105	57 12	15 48	94 01 44	117 21 35	69 14 18	59 34 29	267 25 51	164 56 49
106	56 45	15 38	104 14 31	117 19 07	79 24 54	60 20 19	267 33 30	164 59 17
107	56 09	15 30	118 59 51	117 15 34	93 59 45	61 26 32	267 44 32	165 02 52
108	56 07	15 29	119 48 13	117 15 22	94 47 42	61 30 09	267 45 09	165 03 04
109	54 49	15 06	159 48 39	117 05 43	134 27 50	64 29 43	268 15 06	165 12 43
110	54 48	15 06	160 15 08	117 05 37	134 54 06	64 31 41	268 15 25	165 12 48
111	54 28	15 01	174 05 54	117 02 17	148 37 50	65 33 50	268 25 47	165 16 10
112	55 13	15 15	141 26 48	115 43 21	113 12 29	90 02 59	272 30 58	166 35 08
113	54 46"	15' 05"	155° 08' 50"	115° 40' 02"	126° 47' 46"	91° 04' 28"	272° 41' 02"	166° 33' 29"
114	54 46	15 06	155 17 40	115 40 00	126 56 31	91 05 08	272 41 09	166 38 31
115	54 25	15 00	168 52 39	115 36 44	140 24 37	92 06 05	272 51 18	166 41 46
116	54 11	14 55	182 19 10	115 33 29	153 44 19	93 06 25	273 01 22	166 45 01
117	54 01	14 52	195 52 25	115 30 13	167 10 41	94 07 15	273 11 31	166 48 17
118	53 58	14 48	208 34 16	115 27 09	179 46 06	95 04 14	273 21 01	166 51 22
119	55 05	15 12	149 58 29	114 14 29	118 37 25	117 36 56	277 06 35	168 04 07
120	54 37	15 23	162 53 19	114 11 22	131 25 42	118 34 54	277 16 15	168 07 14
121	54 37	15 04	163 10 50	114 11 18	131 43 04	118 36 13	277 16 28	168 07 17
122	54 04	14 52	189 41 07	114 04 54	157 59 54	120 35 10	277 36 19	168 13 42
123	53 59	14 50	217 09 55	113 58 17	185 14 46	122 38 30	277 56 52	168 20 19
124	53 59	14 50	217 32 38	113 58 11	195 37 17	122 40 12	277 57 10	168 20 25
125	54 07	14 51	232 11 28	113 54 39	200 08 42	123 45 56	278 08 07	168 23 57
126	54 18	14 53	244 27 45	113 51 42	212 18 45	124 41 00	278 17 18	168 26 55
127	54 18	14 53	244 46 43	113 51 37	212 37 33	124 42 25	278 17 37	168 27 00
128	54 07	14 52	199 16 20	112 35 47	164 27 38	148 13 53	282 12 55	169 42 54
129	54 03	14 52	213 02 15	112 32 28	178 06 34	149 15 40	282 23 13	269 46 12

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
130	54' 09"	14' 52"	226° 53' 48"	112° 29' 08"	191° 51' 05"	150° 17' 52"	282° 33' 35"	169° 49' 33"
131	54 20	14 53	238 41 17	112 26 17	203 32 35	151 10 47	282 42 25	169 52 24
132	54 38	14 58	252 48 24	112 22 53	217 32 33	152 14 08	282 52 58	169 55 49
133	54 12	14 54	207 35 59	111 06 58	169 40 26	175 46 57	286 48 35	171 11 47
134	54 40	14 59	248 15 29	110 57 10	209 59 22	178 49 28	287 19 00	171 21 36
135	55 31	15 11	274 34 45	110 50 49	236 05 14	180 47 33	287 38 42	171 27 57
136	55 31	15 12	274 57 20	110 50 44	236 27 37	180 49 15	287 38 59	171 28 02
137	56 00	15 13	287 28 53	110 47 43	248 52 49	181 45 28	287 48 21	171 31 03
138	56 33	15 29	301 29 57	110 44 20	262 46 46	182 48 22	287 58 51	171 34 27
139	57 36	15 46	328 54 15	110 37 43	289 57 10	184 51 22	288 19 22	171 41 04
140	58 06	15 54	343 19 35	110 34 15	304 15 11	185 56 06	288 30 09	171 44 32
141	58 30	16 03	355 53 49	110 31 13	316 43 02	186 52 31	288 39 34	171 47 34
142	58 52	16 08	8 53 57	110 28 05	329 36 35	187 50 52	288 49 17	171 50 42
143	58 53	16 11	9 29 51	110 27 56	330 12 10	187 53 33	288 49 45	171 50 51
144	59 53	16 11	9 41 42	110 27 53	330 23 55	187 54 26	288 49 54	171 50 54
145	58 53	16 11	10 00 43	110 27 48	330 42 47	187 55 51	288 50 08	171 51 00
146	54' 18"	14' 53"	229° 21' 20"	109° 34' 55"	188° 11' 08"	204° 20' 17"	291° 34' 17"	172° 43' 55"
147	54 18	14 53	229 40 10	109 34 51	188 10 48	204 21 41	291 34 31	172 43 59
148	54 18	14 53	229 55 43	109 34 47	188 46 13	204 22 51	291 34 43	172 44 03
149	54 30	14 56	242 14 40	109 31 49	200 56 56	205 18 07	291 43 55	172 07 02
150	54 54	15 03	256 45 59	109 28 19	215 22 53	206 23 18	291 54 47	172 50 32
151	55 23	15 09	269 37 25	109 25 13	228 07 47	207 21 00	292 04 25	172 53 38
152	55 25	15 10	270 25 35	109 25 01	228 55 33	207 24 36	292 05 01	172 53 50
153	55 58	15 19	282 42 42	109 22 03	241 06 26	208 19 45	292 14 13	172 56 48
154	55 59	15 19	283 03 43	109 21 56	241 27 16	208 21 19	292 14 29	172 56 53
155	56 38	15 30	296 25 13	109 18 45	254 42 00	209 21 16	292 24 28	173 00 06
156	58 00	15 53	325 45 27	109 11 41	283 47 21	211 32 56	292 46 25	173 07 11
157	58 28	15 54	337 56 55	109 08 44	295 52 37	212 27 39	292 55 34	173 10 08
158	58 29	15 55	338 22 44	109 08 38	296 18 13	212 29 35	292 50 53	173 10 13
159	58 29	16 02	338 23 00	109 08 38	296 18 29	212 29 36	292 55 53	173 10 14
160	58 51	16 03	350 36 43	109 05 41	308 26 00	213 24 29	293 05 02	173 13 11
161	59 08	16 13	3 26 55	109 02 35	321 09 41	214 22 05	293 14 39	173 16 17

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
162	59'08"	16'13"	3°38'41"	109°02'32"	321°21'21"	214°22'58"	293°14'43"	173°16'20"
163	59 08	16 14	3 50 04	109 02 30	321 32 39	214 23 49	293 14 55	173 16 22
164	59 09	16 14	4 12 03	109 02 24	321 54 26	214 25 28	293 15 13	173 16 28
165	59 19	16 16	43 44 56	108 52 52	1 07 15	217 22 58	293 44 49	173 26 00
166	59 19	16 22	45 26 58	108 52 27	2 48 26	217 30 36	293 46 05	173 26 25
167	54 57	15 03	264 22 03	107 53 40	219 52 28	233 53 06	296 29 55	174 19 15
168	54 59	15 04	264 49 55	107 53 34	220 20 05	233 15 11	296 30 16	174 19 21
169	54 59	15 04	265 27 11	107 53 25	220 57 03	283 57 58	296 30 43	174 19 31
170	55 31	15 11	277 25 06	107 56 32	232 43 54	234 51 40	296 39 40	174 22 24
171	55 32	15 12	277 58 58	107 56 23	233 22 28	234 54 12	296 40 07	174 22 33
172	55 34	15 12	278 30 14	107 56 16	233 53 29	234 56 33	296 40 29	174 22 39
173	55 36	15 12	279 18 21	107 56 04	234 41 11	235 00 09	296 41 06	174 22 51
174	56 13	15 23	291 01 24	107 53 15	246 18 17	235 52 44	296 49 52	174 25 41
175	56 18	15 24	292 20 02	107 52 56	247 06 16	235 53 37	296 50 50	174 26 00
176	57 01	15 36	304 41 38	107 49 57	259 51 35	236 54 05	297 00 06	174 28 59
177	57 52	15 51	318 41 41	107 46 34	273 44 32	237 56 55	297 10 35	174 32 32
178	57'53"	15'51"	319°04'32"	107°26'49"	274°07'12"	237°58'38"	297°10'51"	174°32'27"
179	58 34	16 02	331 03 45	107 43 35	256 00 20	238 52 26	297 19 50	174 35 21
180	58 36	16 02	331 42 05	107 43 26	256 38 20	238 55 18	297 20 19	174 35 30
181	59 15	16 15	345 01 45	107 40 13	299 51 15	239 55 07	297 30 17	174 38 43
182	59 45	16 26	359 02 22	107 36 51	310 44 45	240 58 00	297 40 46	174 42 05
183	60 01	16 29	12 04 36	107 33 42	326 40 22	241 56 30	297 50 32	174 45 15
184	60 22	16 29	13 13 44	107 33 25	327 43 55	242 01 41	297 51 24	174 45 31
185	59 35	16 28	53 50 37	107 23 38	8 05 12	245 03 57	298 21 47	174 55 19
186	55 29	15 11	286 30 42	106 27 32	233 47 15	262 28 10	301 15 55	175 51 22
187	57 07	15 39	313 27 44	106 21 01	265 30 37	264 29 08	301 36 06	175 57 59
188	58 00	15 53	326 10 51	106 17 57	278 07 17	265 26 13	301 45 37	176 01 03
189	58 01	15 53	326 26 13	106 17 54	273 22 31	265 27 22	301 45 48	176 01 06
190	60 26	16 37	7 13 54	106 08 04	318 49 30	268 30 27	302 16 20	176 10 57
191	50 53	16 47	34 27 21	106 01 30	345 49 09	270 32 38	302 36 42	176 17 32
192	59 31	16 27	75 55 07	105 51 30	26 55 53	273 38 43	303 07 44	176 27 32
193	58 45	16 14	89 47 01	105 43 09	40 40 45	274 40 57	303 18 07	176 30 53
194	60 38	16 41	16 07 21	104 39 06	324 35 49	296 06 02	306 52 25	177 40 00

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
I95	61' 09"	I6' 44"	28° 15' 29"	I04° 36' 11"	366° 37' 48"	297° 00' 30"	307° 01' 30"	I77° 42' 55"
I96	61 I0	I6 49	28 40 I2	I04 36 05	337 02 I8	297 02 2I	307 01 49	I77 43 0I
I97	6I 25	I6 57	43 26 59	I04 36 3I	35I 4I 35	298 08 4I	307 I2 53	I77 46 35
I98	6I 26	I6 57	44 I2 25	I04 32 20	352 26 33	298 I2 05	307 I3 26	I77 46 46
I99	6I 20	I6 5I	54 42 2I	I04 29 48	2 5I I5	298 59 I2	307 2I I9	I77 49 I8
200	6I I9	I6 5I	54 52 I9	I04 29 46	3 0I 08	298 59 57	307 2I 25	I77 49 2I
20I	6I I9	I6 5I	54 58 05	I04 29 44	3 06 5I	299 00 23	307 2I 30	I77 49 22
202	58 24	I5 55	356 42 29	I03 I6 59	302 I8 II	32I 34 36	3II 07 I9	I79 02 II
203	60 II	I6 34	23 I2 39	I03 IO 36	329 34 54	323 33 32	3II 27 IO	I79 03 35
204	60 I2	I6 34	23 3I 43	I03 IO 3I	323 53 48	323 34 58	3II 27 24	I79 08 40
205	6I 2I	I6 57	5I 5I 09	I03 03 4I	356 53 52	325 42 05	3II 48 36	I79 I5 30
206	6I 24	I6 59	65 40 38	I03 00 2I	IO 4I 2I	326 44 08	3II 58 56	I79 I8 50
207	6I 24	I6 59	66 I7 07	I03 00 I2	II I7 3I	326 46 52	3II 59 24	I79 I8 59
208	59 35	I6 28	I06 47 52	I02 50 26	5I 27 43	329 48 4I	3I2 29 43	I79 28 46
209	58 48	I6 08	II7 39 08	I02 47 49	62 I3 28	330 37 24	3I2 37 5I	I79 3I 23
2IO	58 48	I6 IO	I8 08 I8	I0I 45 00	320 30 30	350 06 28	3I5 52 47	I80 34 I5
2II	58' 49"	I6' IO"	I8° 20' 15"	I0I° 44' 58"	327° 42' 2I"	350° 07' 22"	3I5° 52' 56"	I80° 34' I7"
2I2	58 49	I6 I2	I8 33 I7	I0I 44 55	320 55 I7	350 08 20	3I5 53 06	I80 34 2I
2I3	59 29	I6 I9	30 45 38	I0I 4I 58	333 0I 26	35I 03 07	3I6 02 I4	I80 37 I7
2I4	59 3I	I6 I9	3I I6 34	I0I 4I 50	333 32 06	35I 05 26	3I6 02 38	I80 37 25
2I5	59 32	I6 I9	3I 29 47	I0I 4I 47	333 45 I3	35I 06 25	3I6 02 47	I80 37 29
2I6	60 09	I6 28	44 39 08	I0I 33 37	346 47 53	352 05 28	3I6 I2 38	I80 40 38
2I7	60 IO	I6 34	45 08 37	I0I 33 30	347 I7 07	352 07 40	3I6 I3 00	I80 40 46
2I8	60 48	I6 48	85 29 I3	I0I 23 46	27 I7 I5	355 03 44	3I6 43 I2	I80 50 30
2I9	60 47	I6 48	86 02 43	I0I 28 38	27 50 28	355 II I4	3I6 43 37	I80 50 38
220	59 30	I6 25	53 4I 25	I00 09 33	352 42 57	I9 4I 43	320 48 50	I82 08 27
22I	59 57	I6 33	80 39 I2	I00 03 02	I9 27 04	2I 42 43	32I 09 00	I82 I6 I3
222	59 54	I6 34	96 09 I5	99 59 24	34 49 I5	22 52 I7	32I 20 36	I82 I9 58
223	59 I3	I6 I9	I2I 22 38	99 53 I9	59 49 50	24 45 30	32I 39 29	I82 26 02
224	59 I3	I6 I9	I2I 28 40	99 53 I7	59 55 49	24 45 57	32I 39 34	I82 26 04
225	58 36	I6 07	I34 47 38	99 50 05	73 03 02	25 45 43	32I 49 3I	I82 49 I7
226	59 I4	I6 22	76 26 24	98 37 00	I2 I3 47	48 I9 30	325 35 37	I83 42 25

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
227	59' 14"	16' 22"	76° 50' 55"	98° 37' 14"	12° 38' 05"	48° 21' 20"	325° 35' 36"	183° 42' 12"
228	59 00	16 18	102 43 22	98 31 00	38 12 25	50 17 28	325 54 57	183 48 25
229	58 59	16 17	103 16 58	98 30 52	38 50 44	50 19 58	325 55 22	183 48 34
230	58 48	16 11	116 40 21	98 27 38	52 07 19	51 20 04	326 05 24	183 51 48
231	58 41	16 11	116 50 47	98 27 36	52 17 40	51 20 51	326 05 31	183 51 50
232	58 43	16 12	116 07 41	98 27 46	51 34 56	51 17 37	326 04 59	183 51 40
233	57 30	15 53	134 10 54	95 29 48	63 23 44	106 30 02	335 17 22	186 49 47
234	57 29	15 53	134 24 04	95 29 45	63 36 48	106 31 01	335 17 31	186 49 50
235	56 50	15 41	148 30 48	95 26 21	77 36 22	107 34 21	335 28 05	186 53 15
236	56 18	15 32	160 56 42	95 23 31	89 55 58	108 30 08	335 37 23	186 56 15
237	56 18	15 32	161 13 43	95 23 17	90 12 50	108 31 25	335 37 36	186 56 19
238	55 46	15 21	175 12 17	95 19 55	104 04 19	109 34 08	335 48 03	186 59 42
239	55 17	15 14	189 19 49	95 16 30	118 04 41	110 37 32	336 58 38	187 08 06
240	54 35	14 58	215 01 53	95 10 18	143 32 43	112 42 53	336 17 52	187 09 18
241	54 35	14 58	215 21 13	95 10 14	143 52 53	112 34 20	336 18 06	187 09 22
242	54 33	14 58	216 20 45	95 09 59	144 51 55	112 38 47	336 18 51	187 09 37
243	54' 19"	14' 54"	228° 54' 24"	95° 06' 57"	157° 13' 11"	115° 35' 09"	336° 28' 16"	187° 12' 40"
244	54 08	14 50	242 53 51	95 03 35	171 11 32	114 37 57	336 38 44	187 16 02
245	54 08	14 50	243 09 14	95 03 31	171 26 48	114 39 06	336 38 55	187 16 06
246	54 03	14 43	257 10 59	95 00 08	165 21 26	115 42 04	336 49 25	187 19 29
247	54 32	14 58	210 38 32	93 44 33	136 09 58	139 08 52	340 44 01	188 35 09
248	54 19	14 52	291 16 33	93 25 07	216 07 05	145 10 46	341 44 21	188 54 35
249	54 39	14 57	305 01 25	93 21 48	229 44 58	146 12 28	341 54 39	188 57 54
250	55 11	15 06	194 00 57	92 21 45	116 58 09	164 49 57	345 01 01	189 58 00
251	54 48	15 04	204 16 55	92 19 17	126 48 57	165 36 01	345 08 41	190 00 29
252	54 47	15 04	204 31 28	92 19 13	127 03 23	165 37 07	345 08 52	190 00 32
253	54 47	15 03	204 40 48	92 19 11	127 12 38	165 37 49	345 08 59	190 00 34
254	54 47	15 03	204 51 47	92 19 08	127 23 31	165 33 38	345 09 08	190 00 37
255	54 46	15 03	205 18 08	92 19 02	127 49 39	165 40 36	345 09 27	190 00 43
256	54 45	15 03	205 28 17	92 19 00	127 59 43	165 41 22	345 09 34	190 00 45
257	54 28	14 56	218 23 13	92 15 53	140 48 06	166 39 20	345 19 14	190 03 52
258	54 10	14 49	230 17 30	92 13 00	152 36 20	167 32 45	345 28 09	190 06 45

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
259	54' 10"	14' 49"	280° 43' 01"	92° 12' 54"	153° 01' 38"	167° 34' 40"	345° 28' 29"	190° 06' 52"
260	54' 09	14' 49	280 54 50	92 12 52	153 13 21	167 35 33	345 28 37	190 06 58
261	54' 09	14' 51	281 12 24	92 12 47	153 30 16	167 36 52	345 28 51	190 06 58
262	54' 09	14' 51	281 37 23	92 12 41	153 55 33	167 38 44	345 29 09	190 07 05
263	54' 01	14' 47	244 22 42	92 09 37	166 34 23	168 35 59	345 38 42	190 10 09
264	54' 01	14' 48	245 01 08	92 09 27	167 12 19	168 38 51	345 39 11	190 10 19
265	54' 01	14' 48	245 09 06	92 09 26	167 20 24	168 39 27	345 39 16	190 10 20
266	54' 01	14' 49	245 22 49	92 09 22	167 34 00	168 40 28	345 39 27	190 10 24
267	54' 01	14' 49	246 00 09	92 09 13	168 11 44	168 43 16	345 39 55	190 10 33
268	54' 00	14' 48	259 08 16	92 06 03	181 12 28	169 42 13	345 49 45	190 13 43
269	54' 00	14' 48	260 16 21	92 05 47	182 19 59	169 47 19	345 50 35	190 13 59
270	54' 05	14' 48	272 16 40	92 02 53	194 14 19	170 41 11	345 59 35	190 16 54
271	54' 14	14' 48	285 10 13	91 59 47	207 01 13	171 39 08	346 09 13	190 20 00
272	54' 15	14' 48	285 51 24	91 59 37	207 42 03	171 42 08	346 09 44	190 20 09
273	54' 15	14' 48	285 31 05	91 59 42	207 21 54	171 40 37	346 09 29	190 20 21
274	54' 30	14' 53	298 55 24	91 56 28	220 39 25	172 40 47	346 19 31	190 23 18
275	54' 31"	14' 53"	299° 29' 10"	91° 56' 20"	221° 12' 54"	172° 43' 18"	346° 19' 56"	190° 23' 27"
276	54' 31	14' 53	299 49 12	91 56 15	221 32 46	172 44 28	346 20 11	190 23 32
277	55' 48	15' 14	339 16 01	91 46 44	260 37 34	175 41 50	346 49 43	190 33 04
278	55' 51	15' 19	340 15 51	91 46 30	261 38 54	175 46 15	346 50 27	190 33 17
279	56' 30	15' 31	353 53 20	91 43 12	175 37 33	176 47 28	347 00 40	190 36 35
280	57' 12	15' 43	7 06 07	91 40 01	253 1 13	177 46 46	347 10 33	190 39 46
281	54' 23	14' 54	227 44 00	90 46 43	147 01 31	194 16 58	349 55 40	191 33 01
282	54' 10	14' 50	240 47 38	90 43 40	153 53 31	195 15 34	350 05 27	191 36 11
283	54' 05	14' 48	254 24 40	90 40 23	173 23 39	196 16 42	350 15 38	191 39 27
284	54' 09	14' 49	267 59 26	90 37 07	186 56 31	197 17 38	350 25 48	191 42 44
285	54' 09	14' 49	268 23 28	90 37 01	187 20 21	197 19 26	350 26 06	191 42 50
286	54' 19	14' 49	281 22 33	90 33 53	200 12 51	198 17 43	350 35 49	191 45 58
287	54' 19	14' 50	281 47 59	90 33 47	200 32 04	198 19 37	350 36 08	191 46 01
288	54' 34	14' 56	294 25 47	90 30 44	213 09 28	199 16 18	350 45 35	191 49 07
289	54' 56	15' 02	309 26 24	90 27 07	228 02 28	200 23 40	350 56 49	191 52 44
290	55' 48	15' 18	336 05 28	90 20 42	254 28 00	202 23 16	351 16 46	191 59 10

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
291	54' 13"	14' 52"	276° 09' 21"	89° 08' 20"	191° 59' 41"	224° 49' 58"	355° 01' 20"	193° 11' 36"
292	55 23	15 10	316 13 11	88 58 41	231 43 11	227 49 47	355 31 19	193 21 15
293	57 55	15 57	24 01 59	88 42 20	298 57 35	232 54 08	356 22 04	193 37 37
294	58 43	16 08	50 25 54	88 35 58	325 08 06	234 52 36	356 41 50	193 44 00
295	58 44	16 12	51 02 58	88 35 49	325 44 52	234 55 23	356 42 17	193 44 08
296	54 26	14 54	283 50 35	87 39 41	196 34 23	252 20 10	359 36 31	194 40 19
297	54 27	14 54	284 13 38	87 39 35	196 57 14	252 21 53	359 36 49	194 40 26
298	54 47	15 00	298 19 47	87 36 11	210 56 14	253 25 11	359 47 22	194 43 49
299	55 15	15 07	311 27 54	87 33 01	223 57 42	254 24 08	359 57 11	194 47 00
300	57 06	15 41	351 18 09	87 23 25	263 27 44	257 22 55	0 27 00	194 56 37
301	58 44	16 09	31 57 34	87 13 36	303 46 31	260 25 24	0 57 27	195 06 25
302	59 20	16 20	71 58 31	87 03 57	343 27 10	263 24 59	1 27 24	195 08 05
303	54 47	14 56	305 53 10	86 07 33	215 23 10	280 54 47	4 22 27	196 12 32
304	55 18	15 04	319 04 42	86 04 23	228 28 00	281 53 59	4 32 19	196 15 43
305	55 19	15 04	319 14 02	86 04 20	228 37 15	281 54 41	4 32 27	196 15 46
306	55 58	15 15	332 18 08	86 01 11	241 34 43	282 53 20	4 42 14	196 18 55
307	59' 38"	16' 15"	40° 07' 54"	85° 44' 50"	308° 50' 05"	287° 57' 46"	5° 32' 59"	196° 35' 16"
308	60 01	16 21	79 33 04	85 35 19	347 55 15	290 54 40	6 02 30	196 44 49
309	59 41	16 16	95 26 09	85 31 39	3 40 17	292 05 58	6 14 23	196 48 38
310	56 54	15 30	357 14 37	84 28 22	263 15 56	311 40 58	9 30 19	197 51 48
311	58 39	15 59	22 06 17	84 22 22	237 54 59	313 32 32	8 48 56	197 57 49
312	60 11	16 24	47 25 25	84 16 16	313 01 17	315 26 10	10 07 52	198 03 56
313	60 51	16 35	87 45 29	84 06 32	353 00 53	318 27 12	10 38 04	198 13 40
314	60 51	16 35	88 04 09	84 06 23	353 19 24	318 28 36	10 38 17	198 13 43
315	60 50	16 34	89 17 10	84 06 10	354 31 48	318 34 03	10 39 12	198 14 02
316	60 50	16 34	89 36 23	84 06 05	354 50 51	318 35 29	10 39 27	198 14 07
317	58 52	16 02	51 35 41	81 21 39	311 04 11	9 36 16	19 09 51	200 58 42
318	59 48	16 18	65 26 21	81 18 18	324 47 50	10 38 24	19 20 13	201 02 03
319	60 31	16 29	77 50 16	81 15 19	337 05 27	11 34 03	19 29 30	201 05 02
320	59 12	16 08	73 27 15	79 49 34	329 42 02	38 10 04	23 55 39	202 30 52
321	59 13	16 08	73 38 46	79 49 21	329 53 27	38 10 56	23 55 48	202 30 54
322	60 23	16 27	99 56 06	79 43 11	355 57 27	40 08 55	24 15 28	202 37 15

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
323	60' 23"	16' 27"	100° 15' 52"	79° 43' 06"	356° 17' 03"	40° 10' 23"	24° 15' 43"	202° 37' 21'
324	60 25	16 28	101 10 13	79 42 53	357 10 57	40 14 27	24 16 23	202 37 34
325	60 47	16 34	127 33 02	79 36 31	23 20 23	42 12 51	24 36 08	202 43 56
326	60 47	16 34	128 14 45	79 36 21	24 01 47	42 15 58	24 36 40	202 44 06
327	59 43	16 16	150 23 23	78 04 13	42 56 28	70 51 02	29 22 39	204 16 19
328	59 21	16 10	163 14 42	78 01 06	55 41 19	71 48 44	29 32 17	204 19 36
329	59 20	16 10	163 44 21	78 00 59	56 10 43	71 50 57	29 32 39	204 19 33
330	54 36	14 53	260 26 21	77 37 40	152 03 39	79 04 57	30 45 02	204 42 53
331	59 12	16 08	131 36 09	76 41 56	21 16 12	96 22 25	33 38 01	205 38 40
332	59 12	16 08	131 43 47	76 41 53	21 28 43	96 23 22	33 38 11	205 38 42
333	59 02	16 05	146 03 36	76 38 27	35 36 19	97 27 18	33 48 50	205 42 09
334	58 48	16 01	158 15 27	76 35 30	47 41 58	98 22 03	33 57 59	205 45 06
335	58 47	16 01	172 08 51	76 32 09	61 28 20	99 24 23	34 08 22	205 48 27
336	57 23	15 38	199 10 06	76 25 33	38 15 52	101 25 39	34 28 36	205 54 59
337	57 23	15 38	199 27 40	76 25 34	88 33 17	101 26 58	34 28 49	205 55 03
338	57 21	15 38	200 09 56	76 25 24	89 15 12	101 30 08	34 29 20	205 55 12

339	57' 36"	15' 42"	180° 53' 40"	75° 03' 14"	67° 06' 05"	126° 59' 20"	38° 44' 21"	207° 17' 27"	/183
340	57 08	15 34	193 28 34	75 00 12	79 34 36	127 55 48	38 53 46	207 20 29	
341	57 05	15 33	194 47 37	74 59 53	80 52 59	128 01 42	38 54 45	207 20 49	
342	56 06	15 17	221 13 33	74 53 31	107 05 30	130 00 20	39 14 32	207 27 10	
343	55 09	15 02	247 46 59	74 47 07	133 25 28	131 59 32	39 34 24	207 33 34	
344	55 28	15 07	229 47 01	73 24 38	112 32 00	157 34 26	43 50 23	208 56 08	
345	54 10	14 46	284 00 29	73 11 34	166 17 58	161 37 48	44 30 57	209 09 13	
346	54 09	14 46	284 27 40	73 11 27	166 44 55	161 39 50	44 31 18	209 09 20	
347	54 04	14 44	297 21 30	73 08 21	179 32 12	162 37 43	44 40 57	209 12 26	
348	54 04	14 44	297 36 03	73 08 17	179 46 38	162 38 48	44 41 08	209 12 30	
349	54 04	14 44	297 55 00	73 08 13	180 05 25	162 40 13	44 41 22	209 12 34	
350	56 51	15 29	199 24 47	72 05 10	79 22 33	182 13 49	47 57 04	210 15 41	
351	56 09	15 18	212 36 28	72 01 59	92 27 33	183 13 03	48 06 56	210 18 51	
352	55 33	15 08	225 43 13	71 58 49	105 27 39	184 11 53	48 16 45	210 22 02	
353	54 40	14 54	251 24 51	71 52 39	130 56 14	186 07 12	48 35 58	210 28 12	
354	54 40	14 54	251 38 02	71 52 34	131 09 19	186 08 12	48 36 09	210 28 16	

TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
355	54' 21"	14' 49"	264° 53' 30"	71° 49' 21"	144° 23' 01"	187° 08' 04"	48° 46' 08"	210° 31' 30"
356	54 06	14 45	280 43 54	71 45 33	160 00 25	188 18 47	48 57 56	210 35 18
357	54 00	14 43	292 36 31	71 42 41	171 47 01	189 12 05	49 06 49	210 38 11
358	54 00	14 43	293 04 15	71 42 34	172 14 30	189 14 10	49 07 11	210 38 17
359	55 30	15 07	13 22 41	71 23 13	251 52 12	195 14 35	50 07 16	210 57 39
360	54 57	14 58	246 03 54	70 27 06	122 35 23	212 38 53	53 01 25	211 53 50
361	54 55	14 58	246 55 47	70 26 54	123 26 50	212 42 46	53 02 03	211 54 02
362	54 29	14 51	260 12 58	70 23 42	136 37 16	213 42 24	53 13 000	211 57 14
363	54 11	14 46	274 17 21	70 20 18	150 34 31	214 45 34	53 22 32	212 00 37
364	54 02	14 43	286 55 00	70 17 15	163 05 45	212 42 14	53 32 00	212 03 41
365	53 59	14 43	300 09 58	70 14 03	176 14 00	216 41 42	53 41 55	212 06 53
366	53 59	14 43	300 53 20	70 13 53	176 57 00	216 44 56	53 42 27	212 07 04
367	55 07	15 01	7 33 28	69 57 48	243 03 19	221 44 09	54 32 21	212 23 09
368	56 18	15 20	36 03 07	69 50 56	271 18 31	223 52 02	54 53 40	212 30 01
369	57 00	15 32	49 21 06	69 47 44	284 29 45	224 51 43	55 03 37	212 33 14
370	54 57	14 58	255 20 01	68 58 03	128 44 11	240 16 10	57 37 47	213 22 58
371	54' 30"	14' 51"	268° 27' 35"	68° 54' 54"	141° 45' 05"	241° 15' 05"	57° 47' 36"	213° 26' 06"
372	54 05	14 44	295 47 30	68 48 18	168 51 08	243 17 45	58 08 04	213 32 43
373	54 14	14 47	323 01 55	68 41 44	195 51 44	245 20 00	58 28 27	213 39 18
374	54 29	14 51	337 28 49	68 38 15	210 11 18	246 24 51	58 39 16	213 42 46
375	54 46	14 55	349 11 00	68 35 26	221 47 33	227 47 33	58 48 01	213 45 35
376	55 08	15 01	1 57 26	68 32 21	234 27 30	248 14 42	58 57 35	213 48 41
377	55 09	15 02	2 44 39	68 32 10	235 14 19	248 18 14	58 58 10	213 48 52
378	55 35	15 09	16 06 45	68 28 56	248 36 19	249 18 14	59 01 30	213 52 06
379	57 42	15 43	68 54 18	68 16 12	300 50 24	253 15 10	59 47 42	214 04 51
380	57 48	15 45	71 17 44	68 15 38	303 12 38	253 25 54	59 49 28	214 05 24
381	54 18	14 48	290 28 28	67 22 47	160 32 10	269 49 25	62 33 31	214 56 18
382	54 11	14 46	304 37 48	67 19 22	174 34 19	270 53 06	62 44 07	215 01 44
383	54 28	14 50	330 44 09	67 13 04	200 27 26	272 50 16	63 03 39	215 08 02
384	56 16	15 20	25 15 11	66 59 56	254 30 49	276 54 57	63 44 26	215 21 10
385	56 20	15 21	27 01 58	66 59 30	256 16 41	277 02 56	63 45 47	216 21 36
386	54 20	14 48	325 27 19	65 47 32	192 10 39	299 22 16	67 29 08	216 33 38

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TABLE 2, Continued

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No.	π	h	m	n	g	g'	ω	ω'
387	54' 36"	14' 53"	338° 59' 52"	65° 44' 16"	205° 36' 20"	300° 23' 02"	67° 39' 16"	216° 36' 55"
388	56 04	15 00	353 21 16	65 40 49	219 50 27	301 27 28	67 50 00	216 40 22
389	58 13	15 52	61 30 53	65 24 23	287 25 29	306 33 23	68 41 01	216 56 49
390	58 43	16 00	75 41 19	65 20 58	301 28 44	307 36 59	68 51 37	217 00 14
391	58 24	15 55	59 26 30	63 58 04	282 19 32	333 19 46	73 08 54	218 23 12
392	58 52	16 02	67 52 58	63 56 02	290 41 43	333 57 39	73 15 13	218 25 15
393	59 59	16 20	95 40 53	63 49 20	318 15 32	336 02 24	73 36 01	218 31 57
394	59 59	16 21	96 46 55	53 49 18	318 21 31	336 02 52	73 36 05	218 31 58
395	60 40	16 32	111 15 30	60 51 58	327 37 00	31 03 42	82 46 32	221 29 28
396	61 16	16 41	125 24 33	60 48 33	341 38 52	32 07 12	82 57 08	221 32 54
397	58 56	16 04	197 07 44	60 31 15	52 45 41	37 29 05	83 50 48	221 50 12
398	61 35	16 30	183 37 43	57 40 53	33 17 15	90 19 52	92 39 34	224 40 43
399	59 26	16 12	204 57 01	56 08 57	51 23 07	118 51 15	97 24 57	226 12 45
400	58 56	16 03	218 24 37	56 05 42	64 43 54	119 51 40	97 35 01	226 16 00
401	58 54	16 03	219 05 48	56 05 32	65 24 44	119 54 45	97 35 32	226 16 09
402	57 28	15 40	246 19 07	55 58 58	92 24 14	121 56 55	97 55 54	226 22 44
403	54 46	14 55	304 44 30	52 51 16	144 14 45	180 10 30	107 38 30	229 30 36
404	54 46	14 55	305 05 06	52 51 11	44 35 09	180 12 03	107 38 45	229 30 40
405	56' 38"	15' 26"	245° 09' 16"	51° 38' 50"	82° 07' 06"	202° 38' 46"	111° 23' 20"	230° 43' 06"
406	56 04	15 17	258 37 58	51 35 35	95 28 58	203 39 15	111 33 25	230 46 21
407	54 45	14 55	298 47 25	51 25 54	135 18 30	206 39 31	112 03 29	230 56 03
408	54 26	14 50	312 03 40	51 22 42	148 27 34	207 39 03	112 13 24	230 59 15
409	54 11	14 46	326 33 52	51 29 12	162 50 24	208 44 08	112 24 15	231 02 45
410	56 20	15 21	254 08 32	50 09 51	87 59 11	230 14 48	115 59 29	232 12 09
411	54 01	14 43	334 44 30	49 50 25	167 54 16	236 16 31	116 59 48	232 31 36
412	53 58	14 42	349 34 03	49 46 51	182 36 18	237 23 04	117 10 54	232 35 11
413	55 16	15 04	53 56 45	49 31 19	246 26 20	242 12 00	117 59 05	232 50 43
414	54 57	14 58	290 50 25	48 34 12	121 19 50	259 55 11	120 56 23	233 47 54
415	54 04	14 44	328 36 46	48 25 06	158 47 01	262 44 42	121 24 39	233 57 00
416	54 00	14 43	355 13 51	48 18 40	185 10 36	264 44 10	121 44 35	234 03 26
417	55 55	15 14	70 51 26	46 33 38	257 07 11	297 19 16	127 10 36	235 48 34
418	54 22	14 49	11 31 34	45 21 08	195 14 48	319 48 41	130 55 38	237 01 08
419	55 08	15 01	40 14 57	45 14 12	223 43 36	321 57 35	131 17 07	237 08 04
420	56 06	15 17	65 52 00	45 08 02	249 07 51	323 52 35	131 36 18	237 14 15
421	56 39	15 26	79 54 35	45 04 41	262 54 12	324 54 55	131 46 42	237 17 36

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TABLE 3

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
1	4	290°40'09"	-15°31'.4	306°11'.6	4	8	347°58'38"	-21°46'.9	9°45'.0
	5	297 35 25		313 02.3	5	4	291 28 49	-21 48.7	213 13.5
	6	299 03 38		314 34.0	5	5	298 12 10		319 59.4
	10	303 50 39		319 21.1	6	299 40 37			321 30.0
2	4	290 37 10	-17 06.8	307 41.1	7	338 14 26			0 04.9
	5	297 29 49		314 36.7	8	348 16 54			10 06.1
	7	337 58 18		355 05.7	26	259 18 33			28 18.5
	8	348 11 05		5 17.2	6	1	264 24 20	-22 12.2	286 35.1
3	19	303 41 22	-21 44.2	320 48.9	4	292 42 13			304 50.5
	24	342 51 51		359 53.2	5	298 56 03			321 07.3
	4	291 16 43		313 02.4	6	300 20 51			322 34.9
	5	297 59 28		319 48.6	11	84 09 42			106 22.1
4	6	299 27 12	-21 46.9	321 08.0	13	89 49 51			112 03.3
	7	337 53 49		359 39.5	26	263 20 22			285 33.0
	4	291 20 33		313 07.2	7	2	287 35 49	-21 11.4	308 49.8
	5	298 03 02		319 51.0	4	291 22 16			312 29.3
5	6	299 30 53		321 15.1	5	298 05 43			319 16.8
	7	337 57 24		358 42.5	7	338 05 20			359 15.9

7	8	348°07'44"	-21°11'.4	9°19'.0	10	6	300°53'19"	-20°53'.8	321°53'.8
	24	342 58 35		4 09.1		7	337 21 05		353 13.6
	25	343 03 32		4 20.5		9	11 47 48		32 46.0
	1	262 10 49		285 35.8		10	13 04 13		34 03.6
8	4	292 19 12	-23 26.7	315 50.8	11	83 36 13			104 35.2
	5	298 47 06		322 12.9		12	85 55 55		106 54.0
	6	300 14 16		323 40.6		13	89 16 56		110 16.1
	7	337 43 01		1 14.0		26	264 14 45		285 11.3
9	8	347 33 53	-23 26.8	11 02.5	II	1	269 17 33	+ 3 43.9	265 28.4
	26	261 23 57		234 51.2		2	291 30 47		287 41.3
	1	262 19 48		235 45.1		4	294 29 11		290 43.2
	4	292 24 59		315 49.7		5	300 21 14		296 35.7
10	5	298 52 07	-20 53.8	322 20.0	13	6	301 42 57		297 53.2
	6	300 19 21		323 46.5		7	336 40 57		332 50.9
	7	337 51 17		1 20.5		8	345 38 43		341 48.0
	8	347 36 38		11 01.4		11	81 20 30		77 33.3
11	26	261 31 36		234 53.6	12	83 53 12			80 06.7
	1	265 27 13		236 25.7		13	87 37 33		83 50.3
	4	293 24 0		314 21.6		16	243 11 37		239 24.7
	5	299 32 59		320 34.4		18	233 13 12		234 30.2

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
11	19	306° 15' 46"	+ 3° 48.9	302° 23.3	12	25	339° 45' 58"	+19° 13.5	320° 32.9
	20	193 43 04		189 55.0	13	8	345 17 13	+21 43.3	323 34.2
	21	224 28 29		220 39.9		11	84 45 01		63 06.0
	22	184 00 41		180 11.3		12	87 24 19		65 44.6
	24	340 28 03		336 40.6		13	91 13 05		69 29.5
	25	340 25 21		336 38.1		24	339 49 05		318 06.3
	26	267 45 08		263 58.5		25	339 39 49		318 00.6
12	4	294 19 54	+19 13.5	275 04.7	14	7	336 26 27	+21 43.1	314 42.0
	5	300 20 21		281 10.8		8	345 19 10		323 40.0
	6	301 40 07		282 27.5		11	84 46 05		63 02.3
	7	336 26 42		317 12.8		12	87 25 09		65 39.5
	8	345 18 04		326 05.8		13	91 13 38		69 29.9
	9	9 24 56		350 12.2		24	339 51 12		318 9.0
11	11	83 30 35		64 19.7		25	339 42 02		317 58.6
	12	86 09 36		66 54.1	15	11	88 08 46	+23 23.5	64 43.2
	13	89 59 56		70 43.2		12	90 39 40		67 18.1
	10	10 08 19		350 54.2		13	94 11 41		70 50.6
	24	339 54 06		320 40.6	16	1	267 09 39	-16 09.6	283 19.2
16	4	293° 43' 20"	-16° 09.6	283° 19.2	17	16	243° 29' 08"	+7° 53.5	235° 38.0
	5	299 42 59		315 49.7		19	306 15 30		298 22.4
	6	301 06 33		317 15.6		21	224 55 42		217 02.0
	7	336 46 45		352 55.1		22	184 46 32		176 52.5
	8	305 48 29		321 58.2		24	340 21 48		332 28.0
	26	265 46 11		381 55.0		25	340 17 51		332 25.6
	17	1	+7 53.5	261 28.1		26	267 50 33		259 59.1
17	2	291 31 37		283 38.1	18	1	268 18 29	+20 50.1	247 28.0
	4	294 31 36		286 41.5		2	291 07 14		270 18.5
	5	300 24 35		292 34.0		4	294 21 26		273 33.0
	6	301 45 52		293 49.5		5	300 24 59		279 33.5
	7	336 39 32		328 45.8		6	301 44 45		280 55.1
	8	345 35 07		337 46.8		7	336 39 48		315 50.7
	11	81 34 54		73 42.0		8	345 34 45		324 44.3
	12	84 09 17		76 16.4		9	9 53 53		349 03.8
	9	9 37 16		1 43 9		11	84 33 54		63 43.3
	13	87 55 55		80 00.5		10	10 39 56		349 49.1
	10	10 25 45		2 32.1		12	87 11 23		66 20.9

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
18	13	90°58'06"	+20°50.1	70°08'6	21	1	268°58'46"	+16°14.5	250°44.2	/192
	20	195 47 17		174 56.1		24	339 58 39		323 38.4	
	24	340 09 16		319 18.3		2	291 17 20		275 02.6	
	19	306 00 23		285 10.8		3	290 11 12		273 57.8	
	25	340 01 20		319 13.8		19	306 02 24		289 48.6	
	21	225 04 42		204 14.3		4	294 24 18		278 12.1	
	26	267 05 59		246 17.5		10	10 05 40		353 50.9	
	16	243 05 46		222 17.5		5	300 22 07		284 07.9	
19	2	290 50 39	+22 39.6	267 09.1		17	244 00 20		227 46.6	
	4	294 10 52		271 31.3		6	301 42 15		285 27.5	
	5	300 19 09		277 40.5		21	225 31 02		209 15.8	/192
	6	301 38 35		278 58.1		7	336 28 24		320 12.9	
	7	336 41 35		314 04.5		9	9 21 55		353 07.2	
	11	85 57 22		63 19.6		8	345 19 46		329 04.1	
	12	88 33 50		65 51.6		22	186 20 39		170 04.9	
	13	92 16 55		69 34.7		26	267 36 32		251 24.7	
20	11	91 13 30	+13 29.0	77 44.5	22	4	294 13 26	+22 02.6	272 13.7	
	13	96 41 34		83 12.5		2	290 55 24		268 53.6	
22	5	300°19'50"	+22°02.6	278°22'2	23	6	301°38'58"	+23°08.2	278°32'3	/192
	3	289 50 38		272 46.7		7	336 52 16		313 44.7	
	6	301 39 14		279 39.7		8	345 54 49		322 50.5	
	24	340 03 27		318 03.6		9	10 41 33		247 34.4	
	7	336 36 10		314 33.1		10	11 32 42		348 23.7	
	25	339 54 41		317 49.2		11	86 40 57		63 30.5	
	8	345 31 57		323 28.4		12	89 15 29		66 05.4	
	19	305 51 29		233 49.9		13	92 54 50		69 50.4	
	11	85 23 23		63 24.3		16	242 06 06		218 58.4	
	12	88 00 59		65 58.5		19	305 50 46		282 41.6	
	13	91 46 31		69 44.7		20	155 05 47		171 56.0	/192
	21	224 55 09		202 51.0		21	224 11 17		201 01.0	
	26	266 46 58		244 45.8		22	185 59 22		165 50.5	
	16	242 49 40		220 43.8		24	340 24 38		317 14.8	
23	1	267 15 15	+23 08.2	244 06.4		25	340 16 36		317 05.9	
	2	290 45 34		267 36.4		26	266 15 03		243 09.9	
	4	294 08 34		271 01.6	24	11	87 23 36	+23 25.2	63 58.1	
	5	300 19 22		277 09.5		13	94 08 37		70 44.2	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
24	12	90°38'18"	+23°25.2	67°13.8	25	25	340°22'02"	+22°41.0	317°41.1	/194
	4	298 54 19		270 30.1	26	1	268 43 59	+18 24.9	250 19.8	
	2	290 24 03		266 54.1		4	294 35 44		276 07.8	
	7	337 12 00		313 45.2		5	300 36 27		282 11.2	
	5	300 11 21		276 47.2		6	301 56 52		283 28.1	
	6	301 31 04		278 05.4		7	336 55 13		318 30.2	
25	4	298 20 15	+22 41.0	270 37.8		26	267 26 17		249 02.1	
	5	299 42 30		277 03.8	27	1	266 05 31	+23 03.6	243 02.9	
	6	301 01 59		278 22.8		4	292 38 39		269 40.0	
	7	336 50 52		314 10.6		5	298 55 28		275 48.4	
	8	346 08 03		323 26.9		6	300 08 37		271 04.1	
	9	11 42 55		349 00.6		7	333 31 20		310 28.6	
	10	12 43 10		350 00.1		8	341 49 43		318 43.8	
	11	89 20 01		66 38.4		9	5 34 04		342 32.9	
	12	91 48 56		69 09.7	10	5 50 58		342 51.0		
	13	95 16 03		72 40.0	11	88 58 32		65 51.2		
	19	305 08 43		282 27.8	12	91 59 13		68 54.9		
	24	340 29 54		316 49.2	13	96 07 20		78 04.7		
31	22	186°00'41"	+23°15.0	162°45.4	35	13	90°54'11"	-23°11.0	114°06.8	/195
	26	265 00 37		241 44.9		24	341 33 27		4 43.2	
32	9	14 32 30	- 7 06.2	21 39.3		25	341 39 13		4 51.2	
	11	89 55 04		97 03.2	36	4	290 24 00	+ 5 17.4	285 04.8	
	12	92 05 49		99 10.5		5	297 08 32		291 48.2	
	13	95 04 17		102 10.0		6	298 29 41		293 12.6	
33	10	16 15 54	-12 20.4	28 36.7		7	336 03 06		330 41.0	
	11	89 04 32		101 27.8		8	345 58 19		340 41.7	
	12	91 14 41		103 35.1	37	1	260 15 19	-18 58.5	279 11.9	
	13	94 13 20		106 32.9		4	291 22 00		310 23.8	
34	9	13 12 19	-23 10.6	36 22.1		5	298 01 02		315 59.1	
	10	14 40 42		37 51.2		6	299 28 11		318.22.2	
	11	85 12 37		108 22.2		7	337 37 49		356 34.3	
	12	87 27 12		110 36.0		8	347 34 49		6 36.4	
35	7	337 02 33	-23 11.0	0 12.1		26	259 45 55		278 43.0	
	8	346 42 18		9 56.1	38	2	288 38 12	-21 33.4	310 09.4	
	11	85 23 41		108 32.6		4	292 12 31		313 45.3	
	12	87 40 36		110 50.4		5	298 45 02		320 18.6	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
27	14	187°41'10"	+23°00'.6	164°38'.8	29	9	14°19'15"	+2°10'.3	12°17'.5
	15	185 41 24		162 37.3		10	15 50 42		13 41.0
	16	242 52 33		219 51.4		11	90 59 00		88 45.3
	19	303 37 19		280 34.3		12	93 11 37		91 01.6
	20	200 01 37		176 59.1		13	96 11 43		94 02.0
	21	226 12 08		203 08.3	30	11	90 27 24	-3 29.0	93 56.5
	26	265 20 46		242 20.1		12	92 38 34		96 09.4
28	4	292 22 20	+19 18.0	273 08.1		13	95 37 05		99 06.2
	5	298 55 42		279 39.9	31	1	265 46 03	+23 15.0	242 31.3
	6	300 16 03		280 58.0		4	293 23 41		270 10.6
	8	346 33 16		327 11.1		5	299 42 21		276 32.1
	7	336 56 52		317 36.2		6	301 00 56		277 50.7
	10	14 09 25		354 50.0		7	336 23 11		313 07.8
	11	90 53 37		71 35.0		8	345 30 23		322 14.0
	12	93 15 28		73 59.8		9	10 44 49		347 28.8
	19	304 23 44		285 07.4		16	241 01 40		217 45.9
	13	96 30 02		77 17.6		19	305 03 20		281 47.6
	9	12 57 00		353 39.3		21	223 25 09		200 09.2
38	6	300°13'00"	-21°33'.4	321°47'.2	41	4	289°36'21"	-26°30'.6	316°05'.8
	7	338 07 34		355 38.5		5	295 58 23		322 29.0
	8	347 58 49		9 31.1		26	233 38 24		290 13.1
	10	15 21 18		36 57.2	42	4	294 59 56	-16 54.8	311 54.3
	26	260 47 30		282 22.7		5	300 56 27		317 51.1
39	4	292 36 47	-22 56.3	315 34.1		6	302 21 30		319 16.2
	5	299 02 15		321 59.4		26	266 50 41		283 43.0
	6	300 29 38		323 25.6	43	2	286 20 50	+11 53.2	274 24.0
	7	337 57 42		0 54.1		4	290 17 19		278 25.2
	26	261 47 20		284 44.0		5	296 53 38		284 56.5
40	4	293 26 59	-23 27.7	316 58.6		6	298 11 03		286 17.9
	5	299 43 39		323 12.7		7	334 24 30		322 32.0
	6	301 10 57		324 42.5		8	343 55 46		332 01.0
	9	13 02 56		36 29.0		9	10 40 00		358 47.3
	11	84 26 15		107 55.0		11	91 39 42		79 50.0
	12	86 41 44		110 07.4		10	11 46 10		359 42.2
	13	89 55 37		113 21.8		12	94 03 43		82 12.1
	26	263 12 56		286 42.6		13	97 17 33		85 25.9

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
43	21	219°45'47"	+11°53'.2	207°56'.8	46	8	345°58'36"	-19°25'.1	5°22'.7
	22	183 31 27		171 38.0		11	85 53 05		105 18.0
	17	237 42 33		224 48.9		26	261 15 12		280 38.7
	24	337 57 09		326 07.8	47	4	289 00 21	-22 40.1	311 41.9
	19	301 59 27		290 06.2		5	296 03 53		318 46.1
	25	337 45 20		325 52.9		7	337 18 24		359 57.2
	26	260 40 27		248 46.7		8	347 45 04		10 26.9
44	4	291 45 20	-20 13.4	311 57.5		26	255 50 31		278 30.5
	5	298 07 32		318 21.0	48	4	296 32 37	-15 49.2	312 18.2
	6	299 31 52		319 43.4		5	302 23 52		318 12.7
	11	85 17 86		105 30.0		6	303 50 08		319 38.7
	26	261 54 33		282 06.6		7	339 37 49		355 22.5
45	11	82 57 06	-17 25.4	100 24.6		8	348 55 20		4 47.6
	13	88 23 32		105 47.0		9	13 20 13		29 11.7
46	4	291 24 32	-19 25.1	310 49.3		10	14 35 48		30 24.0
	5	297 50 59		317 18.3		11	82 51 13		98 44.2
	6	299 15 23		318 40.5		13	88 33 54		104 23.4
	7	336 19 16		355 45.4		24	344 11 31		0 01.2
48	25	344°21'08"	-15°49'.2	0°08'.8	50	25	342°26'36"	+16°36'.0	325°49'.4
	26	268 22 40		284 13.1		22	183 28 50		166 53.4
49	2	292 40 04	-22 49.0	315 34.1	51	4	296 57 51	-19 28.9	316 26.8
	4	295 37 34		318 30.1		5	302 51 52		322 23.3
	5	301 36 12		324 24.2		6	304 19 57		323 49.0
	6	303 03 32		325 57.0		7	340 27 48		359 57.3
	7	339 17 30		2 11.0		8	349 50 44		9 20.2
	8	348 41 41		11 36.5		26	267 58 57		287 29.2
50	4	295 15 50	+16 36.0	278 36.7	52	4	297 25 06	-10 26.5	307 56.9
	5	301 21 47		284 46.2		5	303 16 25		313 43.9
	6	302 44 18		286 08.9		6	304 43 26		315 07.4
	7	338 27 16		321 53.2		7	340 37 14		351 03.7
	8	347 40 46		331 06.8		8	349 56 37		0 22.1
	11	85 40 05		69 05.6		26	268 59 22		279 25.7
	12	88 06 55		71 29.9	53	4	296 02 13	+11 57.1	284 03.6
	13	91 38 08		75 05.7		5	302 11 22		290 15.9
	20	192 35 20		175 58.4		6	303 36 22		291 37.4
	24	342 27 15		325 53.3		7	339 56 25		328 01.1

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
53	8	349°21' 49"	+11°57'1	337°26'0	54	19	305°47'53"	+20°33'2	285°15'1
	9	14 22 54		2 25.6		16	240 22 58		219 50.6
	10	15 38 09		3 38.1		24	341 17 29		319 39.6
	11	86 23 36		74 30.5		25	341 12 18		320 42.2
	12	88 43 55		76 49.8		26	265 01 40		244 32.3
	13	92 05 41		80 6.7	55	11	91 58 57	+15 59.0	76 01.2
	26	266 57 27		255 00.3		12	94 31 28		78 32.1
54		265 49 08	+20 33.2	245 15.0		13	97 55 30		81 57.1
	2	290 21 03		269 47.5	56	11	90 52 07	+ 6 03.9	84 49.4
	4	298 52 30		273 21.8		12	98 27 20		87 22.8
	5	300 11 58		279 43.0		13	96 55 34		90 51.5
	6	301 32 38		281 03.6	57	4	294 16 53	+20 36.5	278 42.5
	7	337 29 23		315 58.7		5	300 40 08		280 03.1
	8	346 48 21		326 14.2		6	302 03 09		281 22.4
	11	88 30 37		67 54.5		7	338 41 54		318 06.1
	12	90 58 02		70 28.7		11	88 43 34		68 07.0
	13	94 24 51		73 49.0		26	264 39 29		244 04.1
	21	222 24 11		201 50.6	58	4	289 54 26	+12 47.3	277 07.4
58	5	296°27'00"	+12°47'3	283°42'9	61	10	12°44'43"	-23°19'5	36°06'3
	6	297 42 25		284 54.5		11	82 56 36		106 13.7
	7	333 09 42		320 22.6		13	88 42 13		112 00.0
	8	342 25 24		329 38.9	62	7	358 10 55	+20 30.4	317 43.4
	11	91 37 58		78 49.3		8	347 48 55		327 17.6
	12	94 07 35		81 19.8		11	89 48 39		69 14.8
	13	97 28 01		84 40.5		12	92 08 54		71 36.2
	14	180 45 06		167 57.8		13	95 23 47		74 54.1
	15	178 38 06		165 52.0	63	4	295 22 34	+15 26.1	279 59.9
	24	326 22 04		323 34.0		5	301 51 25		286 27.5
	25	336 05 50		323 19.6		6	303 18 52		287 52.9
59	11	84 39 52	-18 47.6	103 28.1		7	340 57 50		325 30.6
	12	87 06 07		105 54.1		8	350 45 02		335 17.3
	13	90 32 55		109 21.2		10	17 58 16		2 33.9
60	10	10 46 50	-17 49.2	28 36.2		26	264 17 19		248 53.4
	11	84 43 34		102 32.1	64	1	261 20 40	+23 14.4	238 05.4
	12	87 09 25		104 54.2		2	287 13 11		263 58.9
	13	90 35 35		108 23.9		4	291 07 25		267 51.9

/200

/201

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
64	7	335°53'06"	+23°14.4	312°38.2	66	8	341°13'57"	-7°15.1	348°30.0	/202
	11	91 14 44		67 57.7		26	262 04 41		269 19.8	
	12	98 35 28		70 25.0	67	4	293 51 45	+19 45.1	274 08.5	
	13	96 46 28		73 29.3		5	300 31 06		280 41.7	
	26	261 05.20		237 50.4		7	339 58 14		320 12.8	
65	7	332 25 09	- 138.3	334 03.1		8	349 53 38		330 07.3	
	22	184 18 42		185 56.8		24	344 38 28		324 55.0	
	8	341 24 22		348 02.3		26	262 30 36		242 44.8	
	9	6 40 12		8 20.0	68	2	289 56 43	-22 19.6	312 17.7	
	11	87 13 05		88 47.0		4	293 02 23		315 25.5	
	12	89 50 36		91 30.0		5	299 00 50		321 20.1	
	13	93 27 13		95 05.4		6	300 22 35		322 46.5	
	24	335 34 32		337 18.2		9	9 01 48		31 22.1	
	25	335 19 35		336 57.6		7	335 36 03		357 57.6	
66	4	290 01 55	- 7 15.1	297 17.0		19	304 53 10		327 13.1	
	5	296 19 28		303 33.8		8	344 39 10		6 54.0	
	6	297 36 09		304 51.0		10	9 56 36		32 16.2	
	7	332 20 52		339 36.0		11	81 38 17		103 58.6	
68	21	222°18'36"	-22°19.6	244°38.0	71	8	346°58'13"	-21°44.3	9°38.7	/203
	12	84 09 22		106 26.4		9	11 19 40		33 02.1	
	25	389 23 11		1 44.4		10	12 21 17		34 03.8	
	13	87 49 04		110 06.7		11	82 24 28		104 05.9	
	20	191 46 31		214 06.5		12	84 50 44		106 33.5	
	26	265 42 45		288 04.5		13	88 25 13		110 09.7	
	24	389 25 45		0 50.3	72	4	292 57 54	-22 31.3	315 31.3	
69	8	349 23 10	-15 15.0	4 39.8		5	298 53 45		321 25.1	
70	4	292 00 47	-19 10.5	311 11.8		6	300 14 45		322 46.7	
	5	298 02 37		317 09.6		7	335 10 32		357 42.0	
	6	299 22 54		318 31.3		8	344 06 32		6 39.2	
	7	334 21 56		353 32.9		24	339 50 29		0 23.3	
	8	343 19 39		2 29.5		25	338 45 45		1 13.3	
	26	264 47 07		263 58.4		26	266 05 55		288 37.5	
71	4	295 10 06	-21 44.3	316 57.9	73	4	294 53 05	-22 36.5	317 33.5	
	5	301 03 36		322 49.9		5	300 44 59		323 20.2	
	6	302 26 57		324 08.2		6	302 07 42		324 42.0	
	7	337 50 09		359 34.2		7	337 19 13		359 57.9	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}	
73	8	346° 22' 55"	-22° 36.5	9° 01'.8	76	4	296° 25' 33"	-5° 31'.8	301° 58'.0	/204
	26	267 50 33		290 29.0		5	302 32 25		308 03.6	
74	2	291 07 52	-23 34.4	314 40.6		6	303 57 58		309 33.1	
	4	294 05 41		317 39.4		7	340 17 23		345 48.0	
	5	299 57 25		323 30.3		21	221 17 35		226 51.1	
	6	301 19 18		324 51.0		8	349 42 39		355 15.4	
	7	336 19 09		359 51.8		20	189 36 36		195 08.5	
	8	345 17 49		3 50.0		11	86 06 01		91 40.6	
	24	340 08 15		3 43.4		16	240 44 47		246 16.9	
	25	340 06 04		3 40.1		12	88 25 29		93 57.5	
	26	267 19 59		290 54.7		19	308 49 28		314 22.4	
75	295	295 19 58	-21 26.5	316 43.9		13	91 46 46		97 19.9	
	5	301 11 29		322 37.2		22	180 43 04		186 23.4	
	6	302 34 31		324 01.2		26	267 21 35		272 53.0	
	7	337 49 19		359 16.5		24	344 44 50		350 18.1	
	8	346 54 29		8 22.8		25	344 51 26		350 22.7	
	26	268 13 05		289 43.1	77	9	11 05 25	+17 32.5	353 32.8	/205
76	2	293 16 01	- 5 31.8	298 46.4		10	12 21 16		354 50.0	
77	11	91° 09' 14"	+17° 32'.5	73° 38'.1	79	11	89° 01' 16"	+11° 40'.6	77° 20'.6	
	12	93 28 14		75 55.2		12	91 15 52		79 35.0	
	13	96 34 55		79 03.3		13	94 25 16		82 43.6	
78	2	292 53 17	+ 1 04.8	291 47.0		24	345 05 00		333 23.1	
	4	296 11 25		295 04.4		25	345 12 27		333 32.2	
	5	302 27 15		301 26.1		26	263 58 25		252 16.4	
	6	303 53 41		302 43.3	80	2	289 14 32	+19 35.8	269 39.6	
	7	340 45 35		339 36.0		4	293 02 50		273 26.8	
	8	350 19 50		349 18.1		5	299 43 29		280 07.7	
	26	266 15 06		265 09.2		6	301 08 38		281 34.1	
79	1	264 40 22	+11 40.6	252 59.2		7	339 07 14		319 32.7	
	2	291 19 00		279 37.9		8	349 03 47		329 27.6	
	4	294 53 08		283 12.1		9	15 33 52		355 56.1	
	5	301 23 22		289 35.8		11	90 13 50		70 37.6	
	6	302 49 54		291 08.4		12	92 27 40		72 53.4	
	7	340 25 48		328 45.6		13	95 32 54		75 56.8	
	8	350 13 03		333 34.4	81	7	336 11 20	+23 17.8	312 51.9	
	9	16 04 03		4 22.3		8	346 04 19		322 46.4	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
81	11	90°50'33"	+ 23°17'.8	67°30'.4	84	7	336°47'13"	-22°26'.5	359°10'.8	/206
	12	93 07 09		69 50.0		8	345 43 17		8 15.6	
	13	96 12 49		72 54.9	85	2	292 06 14	-17 14.6	309 20.6	
	24	340 18 59		307 00.6		4	295 13 09		312 32.5	
	25	340 15 31		316 58.3		5	301 13 00		318 30.1	
82	7	335 15 10	+ 21 32.1	313 42.7		6	302 35 01		319 50.1	
	8	345 02 44		323 31.3		7	337 55 36		355 08.8	
	11	90 36 06		69 08.2	19	307 06 44		324 25.4		
	12	92 54 32		71 26.3	8	347 01 23		4 15.7		
	13	96 02 16		74 32.2	9	11 29 58		28 44.5		
	24	339 13 29		317 40.0	11	84 09 58		101 23.8		
	25	339 07 43		317 35.4	16	242 47 04		260 04.3		
83	11	89 01 29	+ 14 07.8	74 53.3	12	86 39 46		103 53.1		
	12	91 26 35		77 15.8	26	267 41 42		284 57.8		
	13	94 44 39		80 37.0	13	90 17 10		107 35.9		
84	4	294 36 48	- 22 26.5	317 00.7	21	224 05 02		241 18.8		
	5	300 30 47		322 56.7	86	4	293 40 52	+ 0 17.2	293 24.0	
	6	301 52 01		324 17.8	5	299 56 01		299 39.1		
86	6	301°14'41"	+0°17'.2	300°54'.2	88	6	301°17'17"	+23°05'.8	278°08'.4	/207
	7	336 26 28		336 10.1		11	82 48 08		59 43.1	
	8	345 29 05		345 11.4		12	85 10 14		62 01.1	
	11	87 53 14		87 36.0		13	88 36 13		65 34.2	
	12	90 28 31		90 11.1	89	4	293 58 04	+ 23 29.4	270 32.6	
	13	94 06 37		93 49.9		5	299 54 30		276 22.8	
87	4	292 47 23	+10 56.3	281 50.3		6	301 17 49		277 48.9	
	5	299 08 51		288 10.0		11	81 53 55		58 23.5	
	6	300 25 59		289 35.4		12	84 20 03		60 53.0	
	7	335 34 15		324 39.6		13	87 53 18		64 24.2	
	8	344 37 29		339 42.2	90	4	294 01 44	+22 19.9	271 42.6	
	11	89 45 34		78 49.7		5	299 54 59		277 39.2	
	12	92 21 09		81 34.0		6	301 17 15		278 56.8	
	13	95 55 50		85 00.7		7	336 25 46		314 06.3	
	24	338 50 18		327 53.9		11	81 14 37		58 49.6	
	25	338 36 57		327 41.8		12	83 44 50		61 28.0	
88	4	293 50 34	+23 05.6	270 43.4		13	87 25 07		65 05.9	
	5	299 52 42		276 46.9	91	15	182 28 13	+11 13.7	171 12.6	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
91	14	180°02'58"	+11°13'.7	168°51'.1	94	6	301°31'46"	+12°55'.3	288°35'.8
92	2	291 11 57	+13 34.4	277 38.2		11	31 52 03		68 58.0
	4	294 14 14		280 39.6		12	84 29 02		71 32.2
	5	300 08 30		286 31.4		13	88 18 43		75 28.0
	6	301 29 06		287 54.7	95	7	336 20 52	+12 55.1	323 28.4
	11	81 32 19		67 52.7		14	181 08 39		168 14.2
	12	84 09 44		70 34.8		15	178 41 01		165 45.8
	13	88 00 28		74 26.3		24	339 55 59		327 00.2
93	2	291 11 43	+13 18.5	277 55.1		25	339 50 08		326 53.4
	4	294 14 29		280 56.6	96	4	294 46 21	+13 16.3	281 27.5
	5	300 09 09		286 50.6		5	300 44 48		287 26.5
	6	301 29 44		288 10.6		6	302 05 58		288 52.0
	7	336 15 52		322 55.1		11	83 23 29		70 10.5
	8	345 07 38		331 44.6		12	85 56 48		72 44.1
	11	81 40 03		68 20.5		13	89 39 45		76 22.5
	12	84 17 21		70 57.1	97	7	337 11 15	+13 20.8	323 51.5
	13	88 07 47		74 52.0		8	346 10 56		332 49.4
94	5	300 11 09	+12 55.3	287 16.0	98	11	91 32 17	-23 16.2	114 48.0
98	12	93°52'29"	-23°16'.2	117°08'.1	101	12	92°23'12"	-21°24'.9	113°49'.8
	13	97 02 35		120 48.0		13	95 47 53		117 11.0
99	1	268 15 30	+ 3 36.9	264 41.3	102	5	297 58 59	- 23 16.6	321 15.0
	4	294 31 00		290 54.0		6	299 18 35		322 34.8
	5	300 35 12		296 59.8		7	336 04 25		359 21.2
	6	301 55 30		298 17.5		8	345 44 18		9 03.0
	7	337 01 24		333 20.0		11	91 24 12		114 40.4
	8	346 00 52		342 21.2		12	93 45 11		117 03.1
	26	267 03 26		263 25.4		13	96 56 44		120 13.2
100	4	294 13 49	- 2 29.0	296 43.0	103	11	91 22 23	-22 44.1	114 06.6
	5	300 23 07		302 52.0		12	98 40 49		116 25.8
	6	301 42 53		304 10.8		13	96 48 31		119 31.5
101	4	292 56 03	-21 24.9	314 21.1	104	4	294 08 12	- 3 54.4	298 03.1
	5	299 21 10		320 44.8		5	300 16 31		304 11.9
	6	300 40 21		322 05.6		6	301 35 52		305 28.2
	7	336 34 56		357 57.3	105	4	294 10 55	- 4 08.1	298 19.2
	8	345 54 47		7 21.0		5	300 19 22		304 25.3
	11	89 55 10		111 18.6		6	301 38 52		305 47.4

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/20

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
106	4	293°49'24"	-8°40'2	302°30'2	109	12	93°37'34"	-22°45'3	116°19'1	/210
	5	300 01 28		308 41.4		13	96 56 36		119 37.6	
	6	301 20 08		309 59.8		26	262 34 28		285 23.3	
107	2	289 54 08	-13 57.1	303 54.4	110	1	262 57.45	-22 46.6	285 40.0	
	4	293 26 44		307 22.7		4	291 58 08		314 44.1	
	5	299 44 14		313 41.5		5	298 29 28		321 16.1	
	6	301 02 39		314 57.1		6	299 47 58		322 31.6	
108	2	289 54 40	-14 08.4	304 01.1		11	91 08 21		113 56.4	
	4	293 27 25		307 35.5		12	93 34 02		116 22.5	
	5	299 45 12		313 51.0		13	96 52 50		119 41.6	
	6	301 03 45		315 09.9		26	262 34 09		285 14.4	
109	1	262 57 52	-22 45.3	285 45.5	111	1	261 58 22	-23 29.2	285 31.4	
	4	291 56 59		314 41.0		4	291 26 43		314 58.6	
	5	298 28 16		321 13.4		5	298 02 04		321 31.1	
	6	299 46 38		322 29.6		6	299 21 02		322 51.7	
	7	335 52 52		358 37.5		7	335 51 40		359 18.0	
	8	345 18 57		8 08.8		8	345 26 39		8 55.1	
	11	91 11 38		113 53.5		11	91 25 18		114 58.1	
111	12	93°47'59"	-23°29'2	117°15'9	115	1	262°21'38"	-23°22'9	285°44'7	/211
	13	97 01 53		120 28.6		2	287 39 44		311 03.4	
	26	261 40 38		285 12.9		4	291 31 13		314 53.9	
112	1	264 17 46	-20 12.8	284 32.4		5	298 04 23		321 29.6	
	2	288 47 45		308 58.6		6	299 22 34		322 46.8	
	4	292 30 49		312 40.2		7	335 33 32		358 51.9	
	5	298 55 57		319 06.2	116	1	261 28 41	-23 22.4	284 51.9	
	6	300 13 28		320 24.4		2	287 08 37		310 29.0	
	26	263 45 19		283 54.5		3	286 07 10		309 41.3	
113	1	263 18 46	-22 21.1	285 40.4		4	291 03 04		314 27.6	
	2	288 13 00		310 31.0		5	297 39 39		321 03.5	
	3	292 14 04		314 38.5		6	298 58 25		322 23.5	
	4	292 00 44		314 20.9		7	335 31 26		358 52.6	
	5	298 30 02		320 49.7		8	345 07 49		8 30.1	
	6	299 47 44		322 08.4		11	91 29 57		114 52.9	
	26	262 53 18		285 16.3		12	93 44 11		117 08.9	
114	14	181 03 44	-21 38.5	202 42.7		13	97 04 54		120 20.0	
	15	178 52 13		200 30.3		26	261 13 27		284 36.3	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
117	1	260°42'07"	-22°22'5	283°03'9	119	2	288°14'46"	-21°58'9	310°10'4	/212
	2	286 43 15		309 07.4		4	292 01 58		314 00.4	
	4	290 39 38		313 07.7		5	298 29 32		320 26.8	
	5	297 19 08		319 41.7		6	299 46 26		321 42.8	
	6	298 38 46		321 00.3	120	3	291 42 12	-23 09.4	314 49.1	
	7	335 34 36		357 56.0		4	291 32 02		314 39.5	
	8	345 18 22		7 41.7		5	298 04 08		321 12.8	
	11	91 15 55		113 34.9		6	299 20 10		322 28.8	
	12	93 35 38		115 57.2	121	3	291 42 31	-23 10.5	314 50.5	
	13	96 44 55		119 09.0		4	291 32 11		314 42.0	
	26	260 29 52		282 53.2		5	298 03 22		321 11.9	
118	3	290 34 55	-20 33.8	311 09.9		6	299 20 29		322 27.9	
	4	290 20 46		310 57.9	122	1	261 03 53	-22 50.8	283 53.0	
	5	297 02 20		317 39.4		2	286 42 57		309 33.8	
	6	298 22 46		318 57.7		3	290 48 51		313 39.2	
	11	90 54 29		111 28.2		4	290 38 50		313 30.3	
	12	93 12 27		113 41.2		5	297 16 01		320 08.9	
	13	96 19 29		116 56.0		6	298 34 19		321 26.7	
122	26	260°50'45"	-22°50'8	283°44'0	125	6	298°18'16"	-15°08'22	313°09'4	/213
123	1	259 57 14	-18 45.0	278 43.3		7	335 53 06		351 09.4	
	7	335 25 01		354 06.1		8	345 46 22		0 54.5	
	8	345 13 35		3 55.8		11	89 31 07		104 39.9	
	11	90 28 53		109 11.7		12	91 47 18		106 57.0	
	12	92 46 57		111 34.4		13	94 53 26		110 00.6	
	13	95 54 26		114 36.7		26	259 28 48		274 36.0	
	14	176 56 08		195 39.8	126	2	286 29 47	-11 20.1	297 53.4	
	15	174 42 22		193 33.4		4	290 21 25		301 45.8	
	26	259 45 20		278 32.9		7	336 07 23		347 29.6	
124	2	286 15 05	-18 42.0	304 54.7		8	346 01 18		357 26.6	
	4	290 11 26		308 56.2		11	88 45 45		100 04.2	
	5	296 52 31		315 34.2		12	91 01 17		102 19.4	
	6	298 13 14		316 57.2		13	94 07 32		105 30.8	
125	1	259 43 25	-15 08.2	274 53.5	127	5	297 03 41	-11 16.0	308 25.4	
	2	286 19 54		301 29.0		6	298 27 06		309 48.9	
	4	290 14 14		305 23.7	128	1	260 36 55	-21 46.6	282 25.1	
	5	296 55 57		312 08.9		2	286 24 35		308 12.0	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}	
128	4	290 ⁰ 21' 10"	-21 ⁰ 46.6	312 ⁰ 12.8	130	12	92 ⁰ 07' 50"	-16 ⁰ 22.9	108 ⁰ 29.3	/214
	5	296 59 28		318 48.3		13	95 15 20		111 39.3	
	6	298 18 11		320 06.3	131	1	259 50 31	-12 36.5	272 26.0	
	7	334 57 28		356 44.2		2	286 16 00		298 53.3	
	8	344 36 35		6 24.1		4	290 08 39		302 49.4	
	14	178 34 10		200 19.3		5	296 48 29		309 27.0	
	15	176 22 32		198 10.1		6	298 10 32		310 50.5	
	26	260 24 45		282 10.0		7	335 35 25		348 14.0	
129	2	286 13 01	-19 30.7	305 45.5		8	345 25 32		358 04.8	
	3	290 23 03		309 52.4		11	89 09 23		101 44.5	
	4	290 09 23		309 44.9		12	91 26 32		104 01.2	/215
	5	296 49 18		316 21.1		13	94 34 15		107 08.5	
	6	298 09 14		317 39.2		14	175 37 01		188 12.3	
	7	335 10 19		354 43.1		15	173 21 06		185 55.9	
	8	344 55 33		4 23.8		26	259 33 26		272 10.0	
	14	177 25 27		196 55.8	132	1	260 07 58	-18 13.7	268 21.8	
	15	175 12 22		194 49.3		2	286 41 45		294 59.7	
130	11	89 50 17	-16 22.9	106 13.6		4	290 29 48		298 47.0	
132	5	297 ⁰ 07' 50"	-18 ⁰ 13.7	305 ⁰ 24.0	134	5	296 ⁰ 59' 46"	-19 ⁰ 48.7	306 ⁰ 51.4	/215
	6	298 31 23		306 47.1		6	298 22 52		309 51.9	
	7	336 05 22		344 25.1		26	259 39 13		269 30.2	
	8	345 56 07		354 04.8	135	1	261 18 40	+10 18.1	260 03.1	
	11	88 06 37		96 19.2		2	287 41 25		287 25.6	
	12	90 22 49		98 35.5		4	291 19 09		291 05.7	
	13	93 30 49		101 42.5		5	297 50 21		297 37.7	
	26	259 44 44		267 59.6		6	299 15 26		299 00.7	
133	1	260 15 06	-20 27.4	280 43.1		11	86 28 52		86 11.3	
	2	286 10 39		306 38.1		12	88 45 07		88 25.1	/215
	3	290 19 08		310 43.1		13	91 55 45		91 32.6	
	4	290 07 28		310 35.0		26	260 41 23		260 23.7	
	5	296 46 39		317 15.9	136	7	336 44 57	+10 24.7	326 17.0	
	6	298 05 48		318 31.3		3	346 30 36		346 07.1	
	26	260 03 21		280 31.2	137	1	262 20 29	+15 38.1	256 42.8	
134	1	260 00 37	-19 48.7	269 45.8		2	288 27 13		282 48.9	
	2	286 31 46		296 18.7		4	291 57 39		286 20.6	
	4	290 21 11		300 12.9		5	298 23 20		292 46.0	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}	
137	6	299 ⁰ 49'01" ⁿ	+15 ⁰ 38.1	294 ⁰ 12.0	139	5	300 ⁰ 27'05" ⁿ	+19 ⁰ 39.0	280 ⁰ 45.5	/216
	7	337 03 08		331 24.9		6	301 53 37		282 17.1	
	8	346 44 31		341 05.0		7	338 16 35		318 36.4	
	11	85 35 08		79 57.8		8	347 42 54		328 04.1	
	12	87 51 40		82 14.4		11	83 17 03		63 36.8	
	13	91 04 22		85 25.5		12	85 34 49		65 54.3	
	26	261 33 12		255 55.4		13	88 54 20		69 14.7	
138	1	263 40 37	+11 06.0	252 35.1		14	172 22 45		152 38.5	
	2	289 26 08		278 18.4		15	169 49 46		150 11.0	
	3	288 13 59		277 07.7	140	11	82 50 58	+22 18.0	60 31.6	
	4	292 47 54		281 46.4		12	85 09 17		62 50.0	
	5	299 06 47		288 01.7		13	88 30 53		66 11.5	
	6	300 33 05		289 27.0	141	11	82 28 16	+23 27.1	58 59.4	
	7	337 32 29		326 26.8		12	84 47 53		61 18.8	
	8	347 08 58		336 00.7		13	88 12 24		64 39.7	
	11	84 41 01		73 37.3	142	11	82 17 08	+23 21.6	58 55.2	
	12	86 57 44		75 48.3		12	84 33 09		61 21.1	
	13	90 12 32		79 06.0		13	88 05 24		64 45.6	
	26	262 41 27		251 34.8	143	11	82 19 58	+23 19.8	59 01.1	
143	12	84 ⁰ 40'52" ⁿ	+23 ⁰ 19.8	61 ⁰ 21.1	147	6	298 ⁰ 04'03" ⁿ	-15 ⁰ 34.8	313 ⁰ 38.5	/217
	13	88 06 55		64 45.1		26	259 32 31		275 09.1	
144	11	82 20 33	+23 19.2	59 01.7	148	1	259 47 06	-15 31.7	275 19.9	
	12	84 41 25		61 18.6		2	286 08 54		301 37.8	
	13	88 08 23		64 50.5		3	290 22 25		305 53.6	
145	11	82 23 01	+23 18.3	59 03.3		4	290 03 08		305 38.4	
	12	84 43 47		61 23.5		5	296 43 28		312 37.3	
	13	88 10 35		64 53.8		6	298 04 55		313 37.3	
146	1	259 47 46	-15 38.3	275 25.8		26	259 32 09		275 05.5	
	2	286 07 01		301 46.7	149	1	259 52 16	-11 46.6	271 40.1	
	3	290 20 02		305 59.7		2	286 19 10		298 08.4	
	4	290 01 23		305 43.9		4	290 10 30		301 57.5	
	5	296 41 34		312 22.2		5	296 49 45		308 36.6	
	6	298 02 50		313 44.3		6	298 12 10		310 04.4	
	26	259 33 00		275 09.8		26	259 33 25		271 23.2	
147	1	259 47 24	-15 34.8	275 21.4	150	5	297 15 10	- 6 51.9	304 04.5	
	2	286 08 07		301 37.8		6	298 39 14		305 30.4	
	4	290 02 24		305 35.0	151	4	291 07 58	- 1 50.4	292 59.9	
	5	296 42 41		312 16.8		5	297 41 19		299 31.2	

TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
151	6	299 ⁰ 06' 19"	-1 ⁰ 50'.4	300 ⁰ 57'.6	153	12	88 ⁰ 09' 33"	+3 ⁰ 32'.1	84 ⁰ 39'.3
	7	336 38 24		338 28.7		13	91 21 00		87 47.0
	8	346 26 20		348 12.2		16	234 49 03		231 22.4
	16	234 22 34		236 17.1		18	229 28 07		225 57.1
	17	235 20 03		237 08.4		19	304 23 37		300 51.6
	18	229 06 03		230 53.3		26	261 11 38		257 39.6
	19	303 42 52		305 34.8	154	14	172 47 46	+3 32.4	175 15.1
	26	260 22 06		262 11.3		15	170 23 02		173 51.1
152	14	175 16 06	-1 36.7	174 53.2	155	1	263 10 12	+9 02.8	254 09.3
	15	170 54 39		172 27.2		2	289 08 57		280 03.4
153	1	261 55 12	+3 32.1	258 24.2		4	292 33 46		283 32.8
	3	292 29 43		288 56.8		7	337 29 49		328 25.2
	4	291 46 20		288 15.8		11	85 00 06		75 57.6
	5	298 13 26		294 42.7		12	87 16 21		78 13.2
	6	299 40 10		296 03.7		13	90 29 52		81 28.7
	7	337 02 14		333 30.8		26	262 15 15		253 14.1
	8	346 46 04		343 10.9	156	1	266 04 17	+18 47.2	247 17.3
	11	85 53 32		82 19.6		2	291 05 16		272 18.4
	2	288 13 15		284 39.2		4	234 13 24		275 31.8
156	5	300 ⁰ 20' 44"	+18 ⁰ 47'.2	281 ⁰ 28'.3	158	19	307 ⁰ 09' 54"	+21 ⁰ 37'.1	285 ⁰ 33'.5
	6	301 47 29		282 55.6	159	7	338 25 30	+21 32.9	316 53.3
	11	83 33 47		64 44.9		8	347 48 48		326 18.8
	12	85 50 57		67 01.5		20	186 30 15		165 00.3
	13	89 09 06		70 17.4	160	2	292 11 25	+23 11.1	269 02.7
	16	237 34 13		218 45.0		4	295 08 16		272 01.3
	17	238 20 53		219 32.9		5	301 03 42		277 56.5
	18	232 02 33		213 16.1		6	302 29 12		279 22.9
	19	306 16 28		288 02.7		7	338 19 51		315 23.1
	26	264 46 19		246 03.3		8	347 37 29		324 23.1
157	7	338 26 56	+18 48.0	319 40.3		11	82 26 52		59 17.3
	8	347 50 46		328 53.3		12	84 47 16		61 36.4
158	2	291 43 52	+21 37.8	270 04.7		13	88 12 35		65 59.6
	4	294 45 17		273 09.3		19	307 25 27		284 15.7
	5	300 45 42		278 06.8		20	187 48 41		164 34.6
	6	302 11 50		280 35.1	161	11	82 08 43	+23 32.2	58.35.8
	11	82 54 50		61 15.4		12	84 30 51		60 57.3
	12	85 13 37		63 37.2		13	87 59 31		64 24.1
	13	88 35 32		66 55.8	162	11	82 09 21	+23 31.8	58 38.9

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
162	12	84 ⁰ 31' 28"	+23 ⁰ 31'.8	60 ⁰ 59'.9	167	6	299 ⁰ 00' 12"	-4 ⁰ 05'.8	303 ⁰ 08'.4
	13	88 00 06		64 30.4		19	303 35 54		307 42.9
163	11	82 10 05	+23 31.7	58 38.4		26	260 07 52		264 13.6
	12	84 32 09		61 00.5	168	1	260 39 04	-3 58.6	264 36.6
	13	88 00 44		64 28.8		2	287 20 17		291 20.7
164	11	82 11 33	+23 31.0	58 40.7		4	291 02 29		295 02.9
	12	84 33 34		60 59.2		5	297 37 46		301 38.9
	13	88 02 02		64 30.3		6	299 02 54		303 02.8
165	11	82 45 37	+16 55.6	65 51.6		19	303 39 31		307 38.6
	12	85 13 39		68 18.9		26	260 08 02		264 05.5
	13	88 50 34		71 52.8	169	1	260 38 55	-3 48.2	264 28.1
166	11	83 00 04	+16 24.1	66 45.6		2	287 23 11		291 12.3
	12	85 27 48		69 03.6		3	291 42 06		295 32.9
	13	89 03 58		72 40.6		4	291 05 14		294 55.0
167	1	260 38 42	-4 05.8	264 42.6		5	297 40 39		301 28.3
	2	287 17 38		291 23.4		6	299 05 59		302 55.5
	3	291 35 46		295 41.8		19	303 43 42		307 33.1
	4	291 00 01		295 06.2		26	260 07 43		263 56.8
	5	297 35 13		301 40.7	170	1	261 29 07	+1 06.6	260 25.8
170	2	288 ⁰ 01' 42"	+1 ⁰ 06'.6	286 ⁰ 53'.7	173	1	261 ⁰ 31' 09"	+1 ⁰ 39'.7	259 ⁰ 51'.1
	4	291 37 29		2 73.6		2	288 12 19		286 30.3
	5	292 08 16		297 05.2		3	292 31 48		290 52.0
	6	299 34 12		298 25.9		4	291 47 27		290 09.0
	19	304 17 48		303 16.3		5	298 18 27		296 40.2
	26	260 49 18		259 39.1		6	299 44 57		298 07.5
171	7	337 11 51	+1 16.5	335 56.4		19	304 31 48		302 55.5
	8	346 58 59		345 46.0		26	260 50 34		259 13.2
	14	172 38 27		171 20.6	174	1	262 37 58	+6 36.9	256 02.6
	15	170 15 18		162 59.1		2	288 56 45		282 19.2
	20	185 28 17		184 13.0		4	292 24 36		285 55.9
172	1	261 31 14	+1 25.3	260 07.7		5	298 49 31		292 14.5
	2	288 08 39		266 46.3		6	300 16 16		293 36.7
	3	292 30 39		291 05.2		7	337 37 23		331 05.2
	4	291 43 56		290 19.5		8	347 19 57		340 41.6
	5	293 14 47		296 48.7		14	172 09 39		165 26.8
	6	299 41 03		298 15.7		15	169 44 02		163 06.9
	19	304 26 39		303 05.6		16	234 56 56		228 30.7
	26	260 50 50		259 24.7		18	229 30 14		222 56.0

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
174	19	305°07'23"	+6°36.9	298°31.9	177	5	300°13'25"	+16°40.6	283°30.7
	26	261 47 20		255 12.1		6	301 40 35		284 57.8
175	1	262 40 48	+6 59.5	255 42.3		19	206 40 51		289 56.3
	2	289 05 00		282 01.0		26	264 15 24		247 37.6
	4	292 32 20		285 35.0	178	11	83 52 00	+16 45.7	67 04.4
	7	337 48 31		350 50.5		12	86 08 07		69 20.4
	16	234 50 37		227 50.4		13	89 24 11		72 35.9
	19	305 17 39		298 17.2	179	4	294 35 12	+20 06.7	274 29.1
	26	261 49 30		254 48.0		8	347 51 30		327 47.3
176	1	264 00 15	+11 55.5	252 01.0		11	83 13 48		63 00.7
	2	289 54 53		278 01.5		16	238 15 22		218 07.6
	4	293 14 15		281 14.8		20	186 05 55		165 59.7
	5	299 32 11		287 33.0	180	4	294 38 44	+20 13.1	74 22.9
	6	300 59 20		289 01.8		7	338 29 59		312 18.7
	19	305 56 01		294 01.6		11	83 14 30		63 01.5
	26	262 57 59		251 02.0		16	238 14 32		217 59.9
177	1	265 29 22	+16 40.6	248 47.8		20	186 01 07		165 49.9
	2	290 51 56		264 08.4	181	1	268 05 22	+22 35.2	245 31.0
	4	294 02 54		277 21.1		2	292 07 40		269 33.4
181	4	295°06'06"	+22°35.2	272°31.5	183	5	301°20'05"	+23°01.6	278°18.2
	5	301 03 40		278 30.1		6	302 44 25		279 43.2
	6	302 29 40		279 54.9		20	188 22 46		164 19.9
	7	338 30 01		315 53.4	184	7	338 17 13	+22 56.0	315 18.8
	8	347 50 23		325 13.5		8	347 28 41		324 31.3
	11	82 43 10		60 03.7		11	82 14 44		59 20.5
	12	85 02 21		62 21.7		12	84 38 11		61 42.5
	13	88 25 28		65 44.5		13	88 08 52		65 12.6
	19	307 28 13		284 53.9	185	11	83 29 50	+13 02.8	70 27.6
	20	187 06 02		164 30.4		12	86 01 04		72 53.0
	26	266 34 06		244 01.3		13	89 41 10		76 35.0
182	2	292 29 05	-23 33.1	268 54.1	186	1	262 17 56	+4 28.8	257 49.6
	4	295 24 14		271 50.2		2	288 52 48		284 24.5
	7	338 26 48		314 53.7		3	293 12 17		288 44.8
	8	347 42 30		324 12.8		4	292 22 27		287 57.5
	19	307 38 26		284 06.4		5	298 49 36		294 20.4
	20	188 24 29		164 50.5		6	300 16 47		295 49.6
183	2	292 34 23	+23 01.6	269 30.1		19	305 09 38		300 43.7
	4	295 28 35		272 30.5		26	261 29 53		257 01.3

TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
187	1	264 ⁰ 59' 11"	+14 ⁰ 37'.6	250 ⁰ 24'.4	190	5	301 ⁰ 43' 13"	+23 ⁰ 26'.6	278 ⁰ 15'.4
	2	290 52 35		276 14.6		6	503 08 41		279 40.1
	4	294 05 47		279 29.9		7	338 51 56		315 22.5
	5	300 19 39		285 41.7		8	348 07 43		324 45.5
	6	301 47 46		287 12.7		11	82 32 52		59 01.1
	19	306 52 25		292 15.7		12	84 53 26		61 22.5
	26	263 48 24		245 14.7		13	88 19 45		64 49.6
188	1	266 24 19	+18 33.3	247 53.2		19	308 06 02		284 38.2
	4	294 46 49		276 11.0		20	188 25 18		165 00.8
	5	300 53 31		282 23.5		26	267 52 00		244 27.4
	6	302 21 19		283 47.6	191	2	292 29 27	+19 13.7	273 13.2
	19	307 27 11		288 48.6		3	296 25 34		277 12.0
	26	265 03 07		246 25.9		4	295 27 38		276 15.2
189	7	339 01 20	+18 36.4	320 25.3		5	301 21 14		282 07.7
	8	348 31 33		329 55.4		6	302 44 25		288 32.6
190	1	269 29 52	+23 26.6	246 08.5		7	338 05 56		318 46.6
	2	292 57 15		269 30.0		8	347 13 18		228 00.4
	3	291 44 03		268 00.0		13	88 46 28		69 34.2
	4	295 50 48		272 25.3		11	82 43 54		63 32.3
191	12	85 ⁰ 10' 45"	+19 ⁰ 13'.7	65 ⁰ 55'.3	194	11	82 ⁰ 56' 38"	+22 ⁰ 56'.3	59 ⁰ 55'.5
192	7	336 47 48	+3 38.4	333 05.0		12	85 15 50		62 19.9
	8	345 46 30		342 10.0		13	88 40 14		65 40.5
	11	85 27 55		81 44.5		19	308 46 04		285 47.9
	12	88 03 48		84 23.5		20	187 59 14		165 05.2
	13	91 46 55		88 09.6		26	268 17 32		245 19.9
193	8	345 22 19	-2 38.8	348 04.6	195	1	270 02 42	+21 02.2	249 00.8
	11	86 51 28		89 32.6		2	293 14 51		272 13.3
	12	89 28 33		92 12.4		4	296 09 04		275 09.4
	13	93 10 49		95 52.9		5	302 01 07		280 59.8
	14	182 30 28		186 07.8		6	303 25 53		282 24.2
	15	180 13 05		183 50.0		16	242 09 02		221 06.8
194	1	269 57 17	+22 56.3	247 02.2		17	242 40 34		221 37.7
	2	293 34 23		270 40.4		18	236 49 51		215 48.4
	4	296 26 34		273 31.5		19	308 18 50		287 19.1
	5	302 18 34		279 22.6		20	189 54 01		168 54.0
	6	303 44 38		280 49.8		26	268 25 42		247 23.3
	7	339 32 21		316 36.2	196	7	339 02 18	+20 58.0	318 01.4
	8	348 49 42		325 52.8		8	348 15 37		327 15.2

TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
196	10	13 ⁰ 49' 04"	+20 ⁰ 58' 0	352 ⁰ 52' 0	199	10	12 ⁰ 18' 59"	+13 ⁰ 00' 3	359 ⁰ 17' 8
	11	82 56 36		61 55.3		20	193 23 22		180 20.9
	12	85 18 55		64 22.7	200	2	292 05 56	+12 56.7	279 06.2
	13	88 47 49		67 50.0		3	296 01 52		283 08.5
197	1	269 32 35	+17 10.1	252 53.0		4	295 11 30		282 17.1
	2	292 41 02		275 33.5	201	11	83 51 20	+12 54.7	70 58.1
	4	295 41 29		278 35.8		12	86 21 10		73 23.0
	5	301 37 27		284 30.1		13	89 58 59		77 08.0
	6	303 00 56		285 54.5	202	4	296 44 50	+23 20.5	273 25.1
	11	83 29 24		66 17.4		5	302 38 32		279 17.4
	12	85 55 05		68 49.7	203	1	270 24 50	+22 13.3	248 09.3
	13	89 27 49		72 22.5		2	294 03 31		271 48.8
	26	268 03 42		250 54.3		4	296 54 11		274 43.8
198	7	338 33 55	+16 59.6	321 33.2		6	304 11 36		281 56.9
	8	347 44 45		330 50.1		16	241 28 33		219 19.6
	20	191 35 07		174 35.6		17	242 04 59		219 49.5
199	7	337 51 16	+13 00.3	324 46.7		18	235 55 34		213 45.5
	8	346 56 27		333 53.6		21	221 06 52		198 53.1
	9	11 22 10		358 22.2		26	263 42 58		246 28.5
3									
204	7	340 ⁰ 00' 49"	+22 ⁰ 11' 1	317 ⁰ 50' 2	206	16	242 ⁰ 06' 18"	+9 ⁰ 32' 0	232 ⁰ 39' 5
	8	349 18 39		327 08.0		17	242 39 16		233 12.7
205	1	269 27 06	+15 02.8	254 25.8		18	237 02 19		227 35.0
	2	292 57 12		277 54.6		21	223 20 01		213 46.3
	4	295 59 24		280 58.6		26	267 19 06		257 48.7
	5	301 57 40		286 58.6	207	1	263 32 10	+9 20.3	258 16.4
	6	303 21 42		288 23.6		2	292 07 11		282 46.4
	7	339 03 53		324 04.7		4	295 17 51		285 57.0
	8	348 18 04		333 20.4		5	301 21 44		292 02.2
	11	84 13 00		69 12.3		6	302 44 18		298 21.0
	12	86 36 52		71 34.9		7	338 22 36		329 00.4
	13	90 06 22		75 03.0		8	347 34 36		338 14.8
	19	308 07 14		293 08.6		11	85 10 13		75 52.4
	20	191 05 24		176 02.4		12	87 37 20		78 14.4
	26	268 00 43		252 58.5		13	91 09 38		81 51.4
206	1	268 34 59	+9 32.0	259 03.0		16	242 02 50		232 46.6
	2	292 07 35		282 34.5		17	242 36 09		233 17.3
	4	295 17 54		285 45.6		18	236 58 32		227 39.6
	5	301 21 25		291 48.1		21	223 16 30		213 59.7

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
207	26	267° 16' 46"	+9° 20.3	257° 52.5	213	19	309° 20' 17"	+21° 00.4	288° 22.9
208	7	336 01 04	-9 46.0	345 47.4	214	7	340 06 22	+20 55.6	319 11.6
	8	345 04 30		354 48.0		8	349 23 48		328 27.7
	11	88 56 58		98 47.0	215	4	297 00 57	+20 53.5	276 10.5
	12	91 32 17		101 17.9		5	302 52 42		281 59.4
	13	95 08 19		104 59.9		6	304 18 54		288 24.0
209	11	89 59 29	-14 17.5	104 39.0		19	309 21 15		288 30.3
	12	92 36 50		106 56.5	216	2	293 44 26	+17 39.2	276 03.0
	13	96 13 20		110 25.2		3	297 45 25		280 05.6
210	4	297 05 52	+22 52.8	274 15.2		4	296 40 49		279 01.2
	5	302 56 23		280 02.2		5	302 35 32		28 54.0
	6	304 23 16		281 28.3		6	304 00 50		286 15.6
	19	309 30 36		286 39.3		7	339 47 51		322 07.7
211	7	340 15 16	+22 51.9	317 23.5		8	349 04 24		331 25.1
	8	349 34 03		326 44.3		16	242 00 28		224 22.2
212	4	297 06 52	+22 50.7	274 16.2		17	242 35 28		224 48.0
213	4	297 00 36	+21 00.4	275 58.0		18	236 37 05		218 57.2
	5	302 51 54		281 51.1		19	308 56 03		291 13.3
	6	304 18 02		283 14.9		21	222 09 09		204 29.4
217	4	296° 40' 22"	+17° 32.7	279° 06.0	219	7	337° 52' 44"	+0° 45.4	337° 04.4
	14	175 29 32		157 55.5		12	89 18 50		88 34.0
	15	172 53 15		155 26.8		20	133 17 09		192 34.3
	19	308 55 58		291 27.2	220	4	296 20 02	+14 47.5	281 36.9
218	1	267 20 16	+0 58.4	266 22.4		5	302 18 31		287 33.0
	2	291 17 08		290 15.5		18	236 38 17		221 55.2
	4	294 37 58		293 40.0		19	308 32 34		293 46.1
	5	300 49 11		299 52.0		21	222 25 21		207 35.5
	6	302 10 34		301 11.0	221	4	295 07 50	+3 45.7	291 25.0
	7	337 52 25		336 50.3		5	301 17 24		297 33.2
	8	347 05 27		346 11.4		6	302 40 00		298 54.0
	11	86 44 52		85 54.5	222	4	294 20 15	-3 20.3	297 42.5
	12	89 13 30		88 17.8		5	300 36 47		303 58.9
	13	92 45 10		91 51.3		6	301 58 15		305 13.5
	16	241 29 07		240 32.9	223	2	289 07 28	-14 10.5	303 21.1
	20	193 19 48		192 19.0		4	292 48 31		306 59.0
	21	223 10 34		222 13.9		5	299 13 17		313 26.7
	26	266 18 17		265 16.5		6	300 31 48		314 41.8
219	4	294 36 33	+0 45.4	293 55.9		16	239 52 19		254 03.4

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
223	17	240 ⁰ 34' 50"	-14 ⁰ 10.5	254 ⁰ 57'.1	226	5	301 ⁰ 17' 58"	+5 ⁰ 28.7	295 ⁰ 45.3
	20	193 57 13		208 03.3		6	302 40 34		297 13.3
	21	222 22 26		236 34.9		19	307 12 23		301 42.1
	26	263 55 17		278 03.5		21	222 47 3		217 18.3
224	7	336 15 17	-14 28.4	350 47.5		26	266 43 32		261 15.3
	8	345 31 38		360 04.3	227	4	295 09 12	+5 10.9	289 53.6
	11	90 04 36		104 35.5		5	301 18 19		295 58.5
	12	92 34 12		107 01.8		6	302 40 57		297 22.6
	13	96 00 39		110 31.4	228	1	265 50 20	-6 36.3	272 25.8
225	4	292 05 26	-18 44.2	310 55.3		2	290 18 37		296 55.4
	7	335 27 21		354 07.7		4	293 50 10		300 27.9
	8	344 42 56		3 28.3		5	300 09 24		306 46.5
	11	91 04 44		109 50.0		6	301 29 52		308 05.5
	12	93 34 52		112 22.1		16	240 28 28		247 05.9
	13	96 59 40		115 45.0		17	241 09 16		247 47.7
	20	194 07 32		212 50.8		19	305 43 52		312 25.4
226	1	267 51 14	+ 5 28.7	262 22.1		21	222 31 50		229 06.4
	2	291 52 50		286 22.8		26	265 02 53		271 35.6
	4	295 09 08		289 42.3	229	4	293 49 22	-6 47.5	300 35.7
229	5	300 ⁰ 08' 58"	- 6 ⁰ 47.5	306 ⁰ 54'.2	231	26	264 ⁰ 06' 25"	-12 ⁰ 29.4	276 ⁰ 34'.8
	6	301 29 28		308 17.3	232	4	293 06 59	-12 29.6	305 39.4
	7	337 24 21		344 12.3		5	299 30 48		311 59.2
	8	346 42 45		353 32.9		6	300 50 08		313 20.1
	14	180 11 32		186 55.0	233	1	263 21 48	-18 43.8	282 07.1
	15	177 52 38		184 40.1		2	288 04 10		306 46.9
230	7	336 43 28	-12 15.5	349 59.9		4	291 52 30		310 38.1
	8	346 02 46		358 15.5		5	298 21 02		317 03.5
	14	180 39 10		192 43.9		6	299 37 48		318 22.5
	15	178 23 33		190 39.0		26	262 56 47		281 40.2
231	231	264 43 44	-12 29.4	277 16.1	234	7	335 09 29	-18 46.7	353 54.8
	2	289 26 45		301 43.5		8	344 23 50		3 07.8
	4	293 05 46		305 32.3	235	1	262 33 34	-21 39.9	284 12.4
	5	299 29 59		311 55.4		2	287 26 39		309 05.2
	6	300 49 22		313 15.4		4	291 17 29		312 02.2
	17	240 32 25		253 03.9		5	297 49 28		319 31.6
	18	234 55 46		247 25.7		6	299 05 55		320 46.8
	19	304 54 45		317 34.9		19	302 50 28		324 30.7
	21	222 10 13		234 38.7		26	262 13 37		283 54.4

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
236	1	261 ⁰ 55' 14"	-23 ⁰ 07.5	285 ⁰ 01.2	238	19	301 ⁰ 55' 51"	-23 ⁰ 29.8	325 ⁰ 10.0
	2	286 53 21		309 56.2		26	261 06 18		284 35.2
	4	290 47 42		313 59.2	239	1	260 57 10	-22 42.9	283 38.8
	5	297 20 12		320 29.8		2	286 10 54		308 54.0
	6	298 36 13		321 45.0		4	290 05 35		312 50.8
	7	334 12 38		357 20.0		5	296 39 45		319 23.0
	8	343 30 58		6 43.3		6	297 56 44		320 42.5
	10	10 50 38		33 59.0		26	260 41 43		283 25.0
	19	302 17 40		325 27.4	240	1	260 39 48	-18 31.7	279 08.7
	26	261 38 52		284 43.5		2	286 00 00		304 29.8
237	14	181 07 28	-23 08.0	204 16.7		4	289 51 11		308 25.9
	15	178 59 17		202 08.2		5	296 24 23		314 58.0
238	1	261 21 16	-23 29.8	284 47.0		6	297 42 47		316 18.2
	2	286 27 43		309 51.4		11	89 38 21		108 12.2
	4	290 22 55		313 51.3		12	92 03 04		110 35.5
	5	296 56 35		320 23.7		13	95 19 50		113 51.2
	6	298 12 55		321 42.0		19	301 38 28		320 06.9
	7	333 59 20		357 31.1		26	260 19 41		278 51.8
	8	343 21 22		6 53.8	241	7	333 59 26	-18 28	352 29.4
241	8	343 ⁰ 29' 10"	- 18 ⁰ 28'	1 ⁰ 58.3	244	5	296 ⁰ 54' 16"	-10 ⁰ 51.1	307 ⁰ 45.9
	10	11 01 31		29 25.5		6	298 15 33		309 10.4
	20	19 04 32		37 30.1		11	87 21 20		98 10.8
	21	218 56 40		237 25.2		12	89 43 27		100 33.1
	22	182 17 54		200 44.1		13	93 00 14		103 50.3
	23	179 36 37		198 08.9		19	302 30 38		313 21.5
242	8	343 38 03	- 18 19.4	1 57.0	245	7	334 56 26	-10 47.7	345 40.4
	13	95 06 25		113 26.0		8	344 30 45		355 16.3
	19	301 45 33		320 02.3		10	11 49 23		22 34.1
	20	190 31 46		209 51.1		20	188 51 40		199 40.3
243	7	334 20 08	- 15 04.2	349 24.6		22	180 24 08		191 13.3
	8	343 52 07		359 00.5	246	7	335 39 08	- 6 00.0	341 38.0
	11	88 30 41		103 36.3		8	345 14 42		351 17.4
	12	90 54 13		105 56.8		11	86 15 32		92 15.0
	13	94 11 01		109 16.5		12	88 36 00		94 33.0
	20	189 52 50		204 53.2		13	91 52 29		97 51.6
	22	181 26 22		196 27.6		22	197 23 31		203 23.8
244	2	286 42 43	- 10 51.1	297 36.8	247	1	260 47 10	-19 21.7	280 05.1
	4	290 24 45		301 19.4		2	285 58 24		305 17.6

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
247	4	289 ⁰ 49'48"	-19 ⁰ 21'.7	309 ⁰ 12'.2	250	26	260 ⁰ 45'50"	-22 ⁰ 01'.0	282 ⁰ 43'.7
	7	333 42 17		353 00.5	251	1	260 54 36	-20 16.9	281 12.4
	8	343 07 58		2 34.4		2	285 48 54		306 06.1
	19	301 32 05		320 54.3		4	289 41 45		309 58.0
	26	260 27 09		279 51.1		6	29 ⁰ 29 29		317 46.1
248	5	299 16 15	+7 28.6	291 48.7		19	301 12 28		321 27.3
	6	300 41 56		293 14.2		26	260 36 04		280 49.9
	7	337 20 12		329 52.7	252	7	333 06 09	-20 15.1	353 23.4
	8	346 50 47		339 17.8		8	342 23 41		2 40.7
	12	86 21 52		78 53.1	253	14	179 52 34	-20 14.0	200 04.0
	13	89 40 11		82 11.8		15	177 43 50		197 58.8
249	7	338 09 22	+12 27.7	325 45.9	254	14	179 50 14	-20 12.7	200 00.4
	8	347 38 00		335 12.4		15	177 41 24		197 54.8
	11	83 32 06		71 02.7	255	7	333 11 45	-20 09.5	353 23.4
	12	85 49 27		73 21.6		8	342 30 21		2 40.7
	13	89 07 43		76 40.9	256	1	260 55 15	-20 08.4	281 05.0
250	1	261 03 10	-22 01.0	283 06.1		2	285 52 08		306 01.2
	4	289 53 14		311 52.9		4	289 44 23		309 59.8
	6	297 41 25		319 43.3		5	296 15 54		316.21.1
256	6	297 ⁰ 32'28"	-20 ⁰ 08'.4	317 ⁰ 45'.9	259	7	333 ⁰ 41'38"	-13 ⁰ 56'.7	347 ⁰ 34'.4
	19	301 17 38		321 27.0		26	260 38 25		274 35.4
	26	260 35 58		280 42.5	260	14	178 04 55	-13 54.2	191 57.2
257	1	260 56 08	-17 24.5	278 22.4		15	175 51 36		189 43.0
	4	289 45 39		307 09.8	261	4	289 54 52	-13 50.4	303 45.5
	5	296 15 40		313 39.5		7	338 45 12		347 33.5
	6	297 33 16		314 56.9		8	343 09 04		356 59.7
	19	301 25 04		318 46.4		19	301 42 12		315 31.8
	26	260 33 01		277 52.8		20	190 40 12		204 28.5
258	4	289 51 15	-14 02.5	303 52.0	262	4	289 56 37	-13 45.5	303 39.4
	5	296 20 23		310 24.0		8	343 12 52		356 59.5
	7	333 38 48		347 39.9		20	190 35 37		204 20.0
259	1	261 06 10	-13 56.7	175 03.6		21	219 01 51		232 44.2
	4	289 52		303 54.5					

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
263	1	261°32'21"	-9°58'8	271°12'1	267	5	296°50'33"	-9°24'6	306°13'0
	2	286 35 23		296 09.0		11	86 53 55		96 21.4
	4	290 15 56		299 55.1		12	89 18 31		98 49.8
	5	296 42 31		306 20.8		13	92 39 11		102 05.4
	16	236 26 48		246 09.5		19	302 23 26		311 46.6
	19	302 11 46		311 52.2		26	261 01 56		270 20.3
	21	218 43 40		228 20.6	268	1	262 17 56	-4 30.2	266 48.2
	26	260 57 35		270 34.2		4	290 56 51		295 27.9
264	7	334 21 18	-9 35.0	343 56.3		5	297 19 50		301 49.1
	8	343 47 48		353 19.5		7	335 07 08		339 39.0
	20	189 45 05		199 21.0		8	344 35 31		349 01.0
265	7	334 22 28	-9 29.5	343 51.2		11	85 51 54		90 17.2
	8	343 49 06		353 17.2		12	88 15 13		92.42.8
	20	189 43 33		199 10.7		13	91 36 02		96 03.1
266	10	10 52 29	-9 27.7	20 15.0		16	236 28 46		240 01.8
267	1	261 38 06	-9 24.6	271 02.4		19	303 03 48		307 36.2
	2	286 44 50		296 09.7		20	188 47 21		193 15.1
	4	290 24 15		299 49.0		21	218 20 50		222 47.3
268	26	261°33'50"	-4°30'2	266°06'6	270	26	262°17'04"	+0°29'5	261°47'7
269	2	287 30 12	-4 19.0	291 49.5	271	2	289 05 39	+5 35.3	283 30.6
	8	344 47 10		349 03.2		4	292 25 28		286 51.9
	13	91 29 52		45.8		5	298 39 47		293 03.4
	22	179 56 20		184 17.1		6	300 04 06		294 30.6
	23	177 11 09		181 28.4		7	336 31 05		330 51.8
270	1	263 10 03	+0 29.5	262 40.7		8	345 58 39		340 18.6
	2	288 11 38		287 40.9		11	84 11 10		78 30.4
	4	291 38 24		291 11.3		12	86 32 04		80 55.6
	5	297 57 19		297 27.7		13	89 53 05		84 20.1
	6	299 20 34		298 46.9		16	237 08 50		231 34.3
	7	335 48 41		335 14.2		19	304 44 36		299 12.7
	8	345 17 05		344 50.6		20	187 29 20		181 56.3
	16	236 44 17		236 17.4		21	218 15 45		212 35.5
	19	303 52 15		303 21.9		22	178 46 10		173 03.6
	21	218 14 15		217 41.3		23	175 58 29		170 24.8
	22	179 24 57		178 50.0	272	2	289 11 18	+5 51.6	283 22.0
	23	176 38 28		176 03.6		4	292 30 56		286 38.7

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
272	5	298°44'40"	+5°51'.6	292°56'.2	273	13	89°51'41"	+6°05.4	33°47'.2
	6	300 09 10		294 17.9		19	304 52 59		298 49.4
	7	336 37 18		330 46.7		20	187 14 37		181 09.7
	8	346 05 11		340 17.1		23	175 45 27		169 40.6
	11	84 09 00		78 18.5	274	2	290 12 04	+10 46.1	279 28.6
	12	86 29 36		80 36.0		4	293 24 07		282 43.7
	13	89 50 19		83 57.2		5	299 33 04		288 48.7
	16	237 08 03		231 13.3		6	300 58 30		290 12.2
	19	304 50 50		298 59.8		7	337 22 16		326 34.5
	20	187 21 32		181 25.0		8	346 48 45		336 03.0
	21	218 12 11		212 20.0		10	13 05 57		2 15.8
	22	178 38 47		172 42.0		11	83 33 25		72 43.0
	23	175 51 09		169 58.1		12	85 52 54		75 03.3
273	4	292 31 44	+6 05.4	286 27.7		13	89 13 38		78 25.1
	7	336 40 52		331 32.4		19	305 48 21		295 01.5
	8	346 09 20		340 08.4		20	186 50 29		176 02.5
	11	84 11 03		78 04.9		22	178 04 30		167 14.9
	12	86 31 22		80 22.4		23	175 15 42		164 26.0

275	2	290°16'56"	+10°55'.1	279°24'.6	278	14	171°46'39"	+21°41'.9	150°05'.7
	4	293 28 34		282 36.7		15	169 08 34		147 28.2
	5	299 37 20		283 42.2	279	8	349 38 30	+23 13.3	326 26.6
	6	301 02 54		290 10.0		10	15 23 58		352 08.2
	11	83 32 42		72 36.3		11	83 01 38		59 48.7
	12	85 51 57		75 55.5		12	85 17 40		62 00.4
	13	39 12 26		78 14.2		13	88 37 36		65 22.8
	19	305 53 40		295 01.2		14	171 47 29		148 36.0
276	7	337 30 39	+11 00.8	326 34.0		15	169 08 07		145 55.1
	8	346 57 31		335 55.8		23	174 08 24		150 53.2
	10	13 14 44		2 17.0	280	10	15 44 13	+23 33.8	952 10.6
	20	186 39 42		175 35.6		11	93 16 30		59 43.5
	22	177 54 35		166 51.3		12	85 32 33		61 58.8
	23	175 05 58		164 04.0		13	88 52 44		65 19.6
277	11	82 50 42	+21 34.9	61 13.2	281	1	261 12 10	-14 48.7	276 01.7
	12	85 07 25		63 32.3		2	286 07 55		300 58.4
	13	88 27 47		66 52.2		4	289 53 33		304 46.6

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TABLE 3, Continued

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No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
281	5	296 ⁰ 22' 01"m	-14 ⁰ 48'.7	311 ⁰ 06'.9	283	7	334 ⁰ 42' 46"m	-6 ⁰ 04'.0	340 ⁰ 46'.2
	6	297 40 24		312 31.8		8	344 08 27		350 13.0
	19	301 37 51		316 24.0		19	302 48 48		308 48.2
	26	260 48 57		275 34.0		21	218 39 50		224 44.9
282	1	261 34 18	-10 46.7	272 25.1		22	180 48 29		186 43.2
	2	286 30 33		297 20.9		26	261 28 01		267 27.4
	4	290 11 28		301 04.2	284	1	262 59 21	-0 56.1	263 57.9
	5	296 37 41		307 25.3		2	287 54 47		288 53.2
	6	297 57 20		308 41.4		4	291 23 10		292 22.6
	16	236 37 27		247 24.4		5	297 42 35		298 38.6
	17	237 22 38		248 16.2		6	299 05 10		300 02.3
	19	302 03 35		312 52.4		19	303 32 32		304 27.6
	21	218 58 30		229 42.0		26	262 03 22		263 07.6
	26	260 59 51		271 45.3	285	7	335 30 33	-0 49.5	336 20.8
283	1	262 10 14	-6 04.0	268 13.9		8	344 57 36		345 47.1
	2	287 07 30		298 11.0	286	1	263 59 38	+4 19.1	259 41.9
	4	290 42 31		296 47.1		2	288 49 26		284 34.3
	5	297 05 45		303 07.8		4	292 10 44		287 52.3
286	5	298 ⁰ 25' 4"m	+4 ⁰ 19.1	294 ⁰ 09'.9	289	13	88 ⁰ 52' 47"m	+14 ⁰ 23.1	74 ⁰ 29'.2
	6	299 49 26		295 29.5		19	306 32 36		292 07.5
	7	336 12 22		331 54.1	290	2	292 54 29	+21 08.2	271 46.2
	8	345 38 48		341 21.0		4	295 49 27		274 40.3
	11	84 17 09		79 56.2		5	301 45 01		280 38.6
	12	86 38 58		82 20.4		6	303 11 50		282 06.6
	13	90 00 56		85 39.6		7	339 14 15		318 10.5
	19	304 26 25		300 11.7		8	348 35 31		327 23.5
	26	262 59 14		258 42.3		11	82 50 19		61 40.4
287	22	179 03 24	+4 26.0	174 37.3		12	85 07 53		64 03.7
288	1	265 07 26	+9.21.2	255 49.3		13	88 29 11		67 27.9
	2	289 47 12		230 22.9		19	308 16 29		287 05.7
	4	293 01 22		293 42.5		20	186 24 36		165 11.4
	26	263 57 28		254 35.2		22	177 28 33		156 25.4
289	2	291 00 32	+14 23.1	276 40.0	291	2	233 29 30	+2 22.3	236 08.7
	4	294 07 04		279 42.4		4	291 52 50		289 31.2
	5	300 11 47		285 48.3		5	293 08 43		295 44.8
	6	301 37 43		287 15.4		6	299 31 56		297 10.2

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
291	19	304 ⁰ 04'47"	+2 ⁰ 22'.3	301 ⁰ 39'.7	294	12	87 ⁰ 26'42"	+16 ⁰ 27'.7	70 ⁰ 59'.8	/242
292	4	294 29 17	+16 41.9	277 48.0		13	90 51 37		74 25.5	
	5	300 30 22		283 48.8	295	9	13 58 15	+15 43.2	358 16.0	
	6	301 56 08		285 18.7		10	15 11 12		359 30.2	
	7	338 02 53		321 20.4		11	85 11 06		69 28.1	
	8	347 24 47		330 45.5		12	87 32 06		71 48.6	
	11	82 53 14		66 05.0		13	90 56 39		75 14.1	
	12	85 12 44		68 25.9	296	1	264 31 09	+5 23.2	259 08.9	
	13	88 35 28		71 49.5		2	289 09 26		288 49.4	
	19	306 51 34		290 12.0		4	292 27 32		287 07.5	
	20	187 03 30		170 19.5		5	298 39 44		293 13.8	
	22	178 07 18		161 30.2		6	300 03 39		294 42.0	
293	11	83 43 01	+22 01.8	61 44.5		19	304 42 30		299 22.3	
	12	86 02 25		64 03.4		26	263 26 22		258 05.2	
	13	89 26 46		67 25.3	297	4	292 29 56	+5 29.5	287 04.0	
294	9	13 54 44	+16 27.7	357 26.1	298	1	265 43 26	+10 42.5	254 59.1	
	10	15 07 17		358 41.8		2	290 17 44		279 37.8	
	11	85 05 33		68 39.4		4	293 28 22		282 43.0	
298	5	299 ⁰ 35'41"	+10 ⁰ 42'.5	288 ⁰ 55'.4	300	11	82 ⁰ 49'03"	+23 ⁰ 10'.0	59 ⁰ 36'.0	/243
	6	301 00 52		290 16.8		12	85 07 34		61 58.0	
	19	305 49 48		295 07.8		13	88 30 58		65 19.8	
	26	264 28 42		253 44.1		16	240 53 10		207 46.5	
299	2	291 16 53	+15 09.5	276 07.2		19	303 44 32		285 36.8	
	4	294 20 59		279 13.0		20	187 23 55		164 12.3	
	5	300 23 23		285 14.6		21	220 34 02		197 26.3	
	6	301 49 14		286 42.1		26	268 07 05		244 59.0	
	7	338 00 00		322 49.1	301	2	293 36 59	+20 33.2	273 04.6	
	8	347 22 58		332 12.2		4	296 35 02		276 00.5	
	19	306 44 33		291 34.8		5	302 31 13		291 55.0	
300	1	269 47 30	+23 10.0	246 34.8		6	303 56 29		282 23.4	
	2	293 30 56		270 18.4		7	339 46 35		319 10.1	
	4	296 22 40		273 15.0		8	349 03 53		329 29.7	
	5	302 14 42		279 04.7		11	84 06 03		63 34.9	
	6	303 41 05		280 27.0		12	86 26 58		65 48.0	
	7	339 31 32		316 24.1		13	89 52 51		69 17.1	
	8	348 49 45		325 39.6		19	308 50 51		288 19.0	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}	
302	11	86°48'00"	+7°33'.1	79°16'.7	306	5	301°47'33"	+20°26'.0	281°22'.7	/244
	12	89 12 44		81 37.5		6	303 14 20		282 48.1	
	13	92 39 20		85 06.8		7	339 14 51		318 46.3	
303	1	266 39 11	+13 15.6	253 23.8		8	348 35 39		328 11.5	
	2	291 05 35		277 46.9		14	172 24 39		151 53.4	
	4	294 10 53		290 58.2		15	169 47 11		149 23.5	
	5	300 14 19		28 03.1		16	239 50 58		219 23.8	
	16	238 30 36		225 19.4		18	234 15 36		213 51.5	
	17	239 12 24		225 56.8		19	308 19 03		287 49.7	
	26	265 17 15		252 02.4		21	219 32 03		199 04.2	
304	4	295 04 42	+17 17.0	277 51.8		26	267 16 44		246 49.9	
	5	301 03 31		283 49.7	307	2	293 27 29	+18 46.7	274 36.6	
	19	307 30 34		290 13.2		4	296 29 10		277 39.6	
305	7	338 38 23	+17 18.8	321 22.1		5	302 28 23		283 41.0	
	8	348 00 42		330 37.4		6	303 53 29		285 06.5	
306	1	268 53 43	+20 26.0	248 23.4		7	339 49 20		320 59.4	
	2	292 58 22		272 32.4		8	349 08 01		330 21.2	
	4	295 52 38		275 30.0		9	13 47 05		354 59.4	
307	11	84°40'39"	+18°46'.7	65°58'.9	310	2	294°36'33"	+23°21'.8	271°17'.7	/245
	12	87 01 45		68 11.2		4	297 25 51		274 09.3	
	13	90 27 08		71 37.3		5	303 18 08		279 58.5	
	16	241 39 40		222 58.2		6	304 46 14		281 26.7	
	18	236 18 07		217 36.9	311	1	270 34 01	+22 32.2	248 01.4	
	19	30 45 31		289 56.5		2	294 42 42		272 09.2	
	20	189 57 30		171 10.6		4	297 34 45		275 05.2	
	21	222 02 04		203 11.2		5	303 28 22		280 57.3	
	22	180 53 29		162 06.0		6	304 55 40		282 21.9	
	23	177 59 53		159 16.3		7	340 56 58		318 27.5	
308	11	87 33 42	+3 49.9	83 45.6		8	350 18 11		327 45.5	
	12	90 00 22		86 15.2		19	310 04 13		287 28.9	
	13	93 28 03		89 47.4		26	268 53 56		246 18.6	
309	11	89 08 33	-3 31.8	92 41.6	312	7	340 11 58	+17 06.4	323 07.4	
	12	90 00 22		86 15.2		8	349 33 33		332 23.8	
	13	93 28 03		89 47.4		10	15 32 42		358 26.1	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
312	14	175°43'39"	+17°06'.4	158°36'.4	315	9	12°32'42"	-0°30'.5	12°04'.1	/246
	15	173 10 10		156 05.1		10	13 37 44		13 02.4	
	22	180 38 05		163 32.2	316	2	290 20 26	-0 37.9	290 58.9	
313	7	337 43 22	+0 09.3	337 35.8		4	293 52 49		294 34.1	
	8	347 03 51		346 50.3		5	300 13 32		300 50.4	
	9	12 30 34		12 18.5		6	301 34 32		302 15.4	
	10	13 35 07		13 27.3		11	88 41 45		89 22.0	
	22	184 04 30		183 54.8		12	91 07 54		91 47.1	
314	2	290 28 46	+0 00.6	290 27.1		13	94 32 44		95 10.2	
	4	293 59 37		293 58.2		19	184 02 02		184 36.0	
	5	300 19 04		300 21.1	317	1	269 17 05	+16 48.5	252 24.8	
	6	301 40 07		301 44.0		2	294 21 24		277 32.3	
	11	88 26 46		88 30.6		4	297 25 10		280 38.3	
	12	90 53 18		90 55.4		5	303 29 48		286 41.6	
	13	94 19 06		94 24.3		6	304 57 25		288 13.8	
	19	305 57 47		305 59.2		7	341 28 51		324 44.1	
315	7	337 39 49	-0 30.5	337 09.7		8	350 57 20		334 10.9	
	8	347 01 25		346 31.0		14	173 55 45		157 07.0	
317	15	171°21'59"	+16°48'.5	154°33'.2	319	5	301°59'27"	+6°49'.7	295°06'.8	/247
	16	240 10 53		223 27.5		7	340 13 52		333 27.2	
	18	234 32 25		217 44.2		8	349 47 36		342 57.9	
	19	310 03 04		293 09.4		9	15 08 22		8 23.0	
	21	220 05 25		203 14.3		14	176 05 32		169 17.5	
	26	267 53 07		261 03.3		15	173 38 20		166 50.0	
318	1	268 01 31	+12 03.6	255 51.1		16	239 22 26		232 36.3	
	2	293 25 14		281 18.6		18	234 00 09		227 11.0	
	4	296 38 10		284 33.4		19	308 12 11		301 21.5	
	5	302 50 01		290 39.5		21	220 19 34		213 23.4	
	6	304 16 55		292 10.9		22	180 48 49		173 59.3	
	7	341 01 45		328 56.4		26	265 52 24		259 01.2	
	8	350 33 51		338 27.5	320	1	267 04 39	+9 00.9	257 00.2	
	19	309 14 47		297 08.6		4	296 04 09		287 06.9	
	26	266 50 13		254 42.6		5	302 21 12		293 18.9	
319	1	266 52 17	+6 49.7	260 03.2		16	239 08 56		230 10.1	
	2	292 20 37		285 32.0		19	308 40 34		299 41.5	
	4	295 41 58		288 49.6		26	266 02 18		259 58.9	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
321	7	340°43'57"	+8°57.2	331°43.0	323	21	219°48'42"	-3°25.0	223°11.5
	8	350 19 29		341 19.0		26	263 48 56		267 11.5
	22	180 08 26		171 09.8	324	4	293 59 47	-3 45.2	297 47.2
322	7	339 02 57	-3 17.2	342 24.5		7	339 04 12		342 53.3
	8	348 43 33		351 56.3		8	348 46 10		352 32.5
	9	14 40 42		17 54.3		16	237 58 24		241 39.0
	10	16 01 01		19 15.4		19	306 25 32		310 08.4
	14	177 08 04		180 24.8		22	181 32 03		185 17.1
	15	174 46 44		178 01.9	325	1	262 23 10	-15 31.5	277 54.1
	22	181 38 05		183 56.3		2	288 08 22		303 40.3
323	1	264 27 21	-3 25.0	267 53.0		4	291 58 38		307 29.5
	2	290 25 32		293 52.7		5	298 33 41		314 06.8
	4	294 02 38		297 29.6		6	299 53 52		315 22.0
	5	300 31 39		303 01.8		11	91 02 24		106 34.2
	6	301 55 35		305 21.5		12	93 23 41		108 54.0
	16	238 06 30		241 28.4		13	96 36 50		122 08.5
	19	306 27 21		309 53.3		19	303 59 52		319 29.1
						26	262 02 18		277 32.4
326	7	336°39'07"	-15°43.5	352°25.3	328	11	91°33'40"	-23°10.0	114°44.6
	8	346 18 03		2 02.3		12	93 58 23		117 05.5
	14	178 49 52		194 32.8		13	97 13 14		120 24.8
	15	176 35 25		192 17.5		14	179 38 23		202 49.4
327	4	290 46 24	-21 30.2	302 18.2		15	177 28 42		200 38.0
	7	335 13 03		336 40.0		16	236 57 09		0 11.2
	8	344 49 20		6 17.5		19	301 56 37		325 07.3
	14	179 10 08		203 39.3		20	191 59 56		215 10.2
	15	176 58 40		198 29.4		21	218 51 43		242 58.9
	19	302 36 32		324 07.1		22	183 37 44		206 49.3
328	1	260 57 22	-23 10.0	284 08.8		26	260 43 51		283 52.7
	2	286 20 13		309 26.7	329	4	290 16 13	-23 11.5	313 27.7
	4	290 16 09		313 29.7		6	298 09 06		321 21.2
	5	296 51 44		320 05.2		7	334 17 55		357 27.3
	7	334 16 53		357 27.8		11	91 30 51		114 42.0
	8	343 46 11		6 58.5		22	183 35 02		206 48.1
	9	10 26 36		33 36.1					

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
330	11	84 ⁰⁰ 2' 58"	-2 ⁰ 53'.2	86 ⁰ 52'.6	334	1	260 ⁰ 53' 55"	-22 ⁰ 41'.1	283 ⁰ 09'.5	/250
	12	86 32 29		89 23.0		2	286 25 53		309 03.0	
	13	90 04 37		92 55.8		4	290 22 22		313 05.0	
331	2	287 40 41	-16 51.0	304 25.2		5	296 59 05		319 37.1	
	4	291 33 50		308 22.5		7	334 35 59		355 17.1	
	6	299 30 44		316 23.8		8	344 09 01		6 49.1	
	16	237 07 58		254 02.0		10	112 02 23		34 34.9	
332	19	303 34 05	-16 54.0	320 29.9		14	179 22 48		203 06.2	
333	1	261 13 00	-20 38.8	281 50.2		15	177 12 27		199 55.0	
	2	286 58 37		307 34.0		16	236 48 30		259 29.5	
	4	290 54 08		311 29.2		19	302 06 48		324 46.5	
	5	297 31 45		318 10.8		21	219 41 12		222 21.5	
	6	298 50 37		319 27.5		22	183 24 05		206 04.2	
	7	335 28 59		356 11.5		26	260 41 20		283 19.6	
	8	345 07 23		5 40.5	335	8	343 24 48	-23 30.8	6 54.1	
	19	302 47 23		323 28.2		14	179 28 34		203 59.7	
	26	260 59 20		281 40.4		15	177 18 54		200 49.0	
335	19	301 ⁰ 40' 12"	-23 ⁰ 30'.8	325 ⁰ 12'.6	337	26	260 ⁰ 42' 37"	-21 ⁰ 18'.5	282 ⁰ 02'.6	/251
	21	219 41 17		243 13.6	338	4	289 48 38	-21 13.6	321 06.3	
	22	183 27 19		206 57.8		7	333 17 58		354 27.4	
336	7	333 12 19	-21 20.7	354 32.2		8	342 36 12		3 51.6	
	8	342 29 51		3 50.1		11	89 17 36		110 29.0	
	10	9 34 35		30 55.1		20	191 46 56		21 07.8	
	20	191 54 34		213 15.5		21	219 44 52		240 59.8	
	22	183 22 27		204 44.0	339	1	260 50 17	-23 18.0	284 10.7	
337	1	261 06 08	-21 18.5	232 28.7		2	285 54 50		309 08.9	
	2	285 57 15		307 11.1		4	289 48 44		313 07.9	
	4	289 46 30		311 03.2		5	296 21 40		319 41.1	
	5	296 16 11		317 36.8		7	333 25 17		356 45.5	
	11	9 36 04		30 56.7		8	342 46 58		60 00.7	
	12	89 25 31		110 41.3		16	236 55 34		260 09.9	
	13	91 54 45		113 19.6		19	301 23 00		324 40.8	
	16	236 69 06		258 21.8		21	219 51 33		243 04.5	
	19	301 22 12		322 39.3		22	183 39 55		206 58.4	
	21	219 47 29		241 03.5		26	260 33 30		283 53.6	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}	
340	1	260°57'59"	-22°07'.4	283°04'.5	341	22	183°25'25"	-21°59'.9	205°26'.3	/252
	2	285 50 38		307 56.3	342	1	261 50 02	-16 29.8	278 30.5	
	4	289 41 50		311 50.5		2	286 15 25		302 49.2	
	5	296 12 31		318 19.2		4	290 00 28		306 33.0	
	7	333 06 25		355 12.7		5	296 24 35		306 52.6	
	8	342 23 46		4 27.9		6	297 42 38		314 11.7	
	9	8 35 29		30 44.9		7	333 16 22		349 42.4	
	10	9 32 44		31 39.0		8	342 29 15		8 53.3	
	14	179 40 15		201 43.6		11	87 20 14		103 49.1	
	15	177 30 57		199 37.5		12	89 51 25		106 06.5	
	16	236 59 49		259 10.4		13	93 19 48		119 44.8	
	19	301 14 45		323 20.1		19	301 39 43		318 11.1	
	21	219 53 07		242 56.3		26	261 14 46		277 42.5	
	22	183 35 59		205 46.1	343	7	334 07 32	-12 52.4	346 59.8	
	26	260 37 10		282 46.5		8	343 20 08		356 12.7	
341	4	289 45 15	-21 59.9	311 48.1		11	84 53 10		97 44.3	
	10	9 40 48		31 37.9	22	181 56 01		194 46.6		
	21	219 48 10		241 46.8						
344	1	262°20'15"	-13°49'.6	276°09'.6	345	2	289°38'48"	+7°31'.8	202°17'.2	/253
	2	286 34 25		300 19.8		4	292 48 46		285 24.8	
	4	290 11 24		304 02.7		5	298 52 13		291 31.5	
	5	296 32 20		310 22.0		6	300 14 58		292 50.9	
	6	297 50 40		311 38.1		7	335 54 24		328 34.4	
	7	333 17 11		346 59.9		8	345 06 50		247 43.5	
	8	342 26 31		356 13.3		9	9 49 26		2 25.4	
	9	7 50 43		21 35.2		10	10 52 17		3 32.1	
	10	8 44 48		22 30.0		11	82 32 39		75 08.8	
	13	92 26 52		116 11.7		12	84 59 21		77 38.0	
	14	178 52 38		192 42.0		13	88 31 14		81 08.8	
	15	176 38 21		190 26.8		19	304 49 59		297 24.5	
	16	237 41 59		251 28.3		22	180 41 11		172 23.7	
	19	301 50 45		315 39.6	346	2	289 42 08	+6 39.8	283 05.2	
	21	220 09 56		233 59.9		10	10 57 07		4 15.7	
	22	182 56 47		196 45.2		12	84 59 02		78 15.9	
	26	261 39 06		275 27.7	20	189 46 25		183 09.1		

TABLE 3, Continued

No.	No. of crater	P _{comp}	χ	P _{obs}	No.	No. of crater	P _{comp}	χ	P _{obs}
347	2	290°37'36"	+11°28.9	279°14.4	350	2	285°56'36"	-21°01.5	306°56.0
	4	298 41 23		282 15.5		4	289 43 48		310 39.4
	5	299 40 25		288 14.0		5	296 10 25		317 13.6
	6	301 04 16		289 38.8		6	297 26 30		318 30.6
	11	82 12 14		70 46.2		19	301 10 27		322 13.4
	12	84 36 58		73 07.4	351	2	286 07 36	-18 20.0	302 26.8
	13	88 07 25		86 43.0		4	289 50 20		308 09.8
	19	305 27 05		294 18.3		5	296 14 11		314 33.1
348	7	336 46 51	+11 31.5	325 16.8		6	297 30 57		315 49.9
	8	346 00 22		334 32.0		19	301 20 14		319 42.7
	9	10 36 09		359 04.6	352	2	286 27 26	-14 49.9	301 17.4
	10	11 41 58		0 09.0		4	290 04 58		304 56.7
	20	189 18 31		177 47.6		5	296 25 39		311 14.6
	22	180 05 01		168 36.1		6	297 43 20		312 35.2
349	4	298 45 31	+11 36.2	282 09.7		19	301 39 22		316 26.0
	8	346 03 35		334 26.8	353	4	290 55 12	-5 56.6	296 52.4
	13	88 07 38		76 28.0		5	297 08 35		302 04.0
	20	189 14 27		177 39.2		6	298 28 13		294 23.7
353	7	333°50'00"	-5°56.6	339°44.0	355	7	334°32'27"	-0°39.5	335°08.8
	8	342 56 02		348 53.7		8	343 40 09		344 16.7
	9	7 53 42		13 51.0		9	8 29 19		9 07.8
	16	238 34 32		244 31.8		11	83 17 45		83 55.6
	19	302 39 14		308 35.3		12	85 48 56		86 26.4
	21	220 41 19		226 38.6		13	89 24 41		90 02.9
	22	182 37 21		188 37.5		16	239 00 56		239 37.3
354	4	290 59 28	-6 01.2	296 54.8		19	303 25 24		304 03.7
	7	333 58 32		339 57.2		20	191 07 40		191 47.1
	8	343 06 01		349 06.3		21	220 48 05		221 27.0
	9	8 05 23		14 04.8		22	182 00 56		182 39.4
	16	238 30 26		244 30.1		26	263 34 38		264 13.3
	22	182 24 18		188 16.9	356	7	335 39 05	+5 20.6	330 22.3
355	1	264 39 12	-0 39.5	265 18.8		9	9 34 07		4 13.8
	2	288 16 06		288 58.6		21	220 43 26		215 22.2
	4	291 35 21		292 15.8	357	1	265 47 57	+9 59.9	256 49.7
	5	297 44 26		298 29.7		2	290 11 24		280 14.0
	6	299 05 14		299 47.0		4	293 17 24		283 20.7

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
357	5	299°17'43"	+9°59'9	289°18'5	358	13	88°11'12"	+10°07'0	78°03'0	/256
	6	300 40 45		290 40.6		20	189 51 42		179 43.2	
	7	336 15 56		326 13.5		21	222 00 08		211 55.8	
	8	345 27 13		335 26.9	359	8	350 24 58	+23 17.3	327 07.4	
	9	10 02 59		0 02.8		9	14 53 42		351 37.0	
	11	82 11 47		72 11.3		11	84 14 30		60 53.1	
	12	84 38 14		74 36.7		12	86 31 09		63 13.7	
	13	88 10 46		78 08.4		13	89 51 29		66 35.8	
	16	240 02 10		230 07.0		14	173 17 28		150 01.0	
	19	305 18 53		295 21.8		22	178 26 26		154 11.4	
	20	189 57 32		179 57.1	360	1	263 35 51	-7 33.8	271 05.0	
	21	221 03 02		211 02.0		2	287 11 56		294 43.8	
	22	180 42 11		170 43.0		4	290 39 34		298 11.3	
	26	265 26 24		255 28.1		5	296 53 14		304 27.7	
358	4	233 20 28	+10 07.0	283 15.1		6	298 11 57		305 45.3	
	7	336 20 06		326 12.1		7	333 19 28		340 52.1	
	9	10 07 51		359 58.0		8	342 20 13		349 52.5	
	11	82 12 53		72 03.4		16	238 46 30		246 19.8	
360	19	302°17'04"	-7°33'8	309°52'2	362	21	221°08'38"	-2°12'5	223°17'5	/257
	21	221 04 48		228 37.5		22	182 41 03		184 51.1	
	22	183 23 00		190 54.1		26	263 27 05		265 37.3	
	26	262 41 35		270 11.3	363	1	265 29 44	+3 18.8	262 11.2	
361	4	290 43 53	-7 20.4	298 03.1		4	292 05 20		288 48.1	
	8	342 28 17		349 48.8		5	298 10 29		294 48.7	
362	1	264 30 09	-2 12.5	266 46.6		6	299 31 45		296 12.3	
	2	287 58 09		290 15.4		7	334 55 13		331 38.0	
	4	291 18 51		293 32.1		8	344 01 35		340 41.3	
	5	297 28 16		299 43.0		9	8 40 43		5 20.9	
	6	298 48 18		301 59.9		14	177 31 25		174 12.8	
	7	334 04 31		336 13.9		15	175 06 06		171 46.6	
	8	343 08 05		345 16.2		16	239 37 20		236 19.5	
	9	7 52 48		10 03.1		19	303 56 34		300 41.3	
	14	178 24 49		180 37.8		21	221 13 17		217 51.1	
	15	176 03 47		178 15.2		22	181 57 54		178 37.3	
	16	239 10 29		241 22.6		26	264 18 12		261 00.2	
	19	303 03 27		305 16.5						

TABLE 3, Continued

No.	No. of cra- ter	P comp	X	P obs	No.	No. of cra- ter	P comp	X	P obs
364	4	252°48'17"	+8°17.6	284°29.7	365	12	84°19.54"	+13°00.0	71°23.1
	5	298 49 33		290 32.0		13	87 54 00		74 53.5
	7	335 37 10		327 20.0		16	240 39 19		227 38.7
	8	344 44 40		336 27.6		19	305 37 37		292 39.3
	9	9 17 53		1 02.1		20	190 22 28		177 23.6
	16	240 08 49		231 53.0		21	221 36 30		208 36.0
	19	304 43 29		296 25.6		22	181 01 14		168 01.8
	20	190 50 36		182 33.0		26	266 00 54		253 02.2
	21	221 26 41		213 08.3	366	4	293 42 55	+13 09.9	280 32.2
	22	181 32 19		173 16.6		5	299 40 23		286 31.6
365	1	267 26 34	+13 00.0	254 25.7		9	10 12 34		357 03.1
	4	293 37 56		280 36.4		11	81 54 58		68 46.7
	5	299 35 31		286 35.8		12	84 21 29		71 10.8
	6	300 58 32		287 56.8		13	87 54 59		74 44.9
	7	336 26 49		323 23.5		16	240 37 57		227 29.4
	8	345 35 59		332 37.3		19	305 43 38		292 35.0
	9	10 04 55		357 06.3		20	190 13 27		177 04.2
	11	81 52 57		68 53.7		21	221 32 16		208 22.8
366	22	180°52'52"	+13°09.9	167°42.5	370	19	302°48'01"	-3°56.7	306°44.2
367	7	340 23 33	+23 29.1	316 52.6	371	1	265 16 41	+1 20.2	263°53.4
	8	349 42 39		376 11.8		2	288 28 57		287 07.2
	11	83 53 23		60 22.6		4	291 44 39		290 23.3
	12	86 11 50		62 42.8		5	297 50 17		296 30.0
	13	8 34 50		66 04.1		6	299 10 45		297 50.7
368	11	85 45 30	+20 28.6	65 16.5		7	334 23 15		333 45.7
	12	88 01 31		67 30.2		8	343 25 20		342 05.8
	13	91 18 56		70 48.8		16	239 45 27		338 23.5
369	11	86 36 41	+17 30.1	69 06.1		18	234 53 55		233 32.5
	12	88 52 04		71 22.0		19	303 30 11		302 12.6
	13	92 07 20		74 36.8		21	221 34 19		220 12.1
370	2	287 45 36	-3 56.7	291 41.1		26	264 06 56		262 48.2
	4	291 07 22		295 03.5	372	2	290 17 17	+11 41.0	278 35.6
	5	297 17 00		301 14.1		4	293 21 19		281 41.5
	6	298 36 29		302 30.5		5	299 19 16		287 41.4
	16	239 16 51		243 13.6		6	300 41 47		288 59.7

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
372	7	336°04'40"	+11°41'0	324°21'8	374	16	241°39'07"	+21°53'6	219°49'6
	8	345 11 39		333 28.9		19	308 01 36		286 06.5
	19	305 17 35		293 36.6		20	189 14 53		167 19.5
373	2	292 06 10	+19 23.7	272 46.2		21	221 48 21		199 56.2
	4	295 02 01		275 37.8	375	2	298 24 53	+23 09.2	270 15.4
	5	300 54 45		281 32.6		4	296 18 02		273 08.7
	6	302 18 54		282 53.8		5	302 09 43		27 59.3
	7	337 49 35		318 18.8		6	303 34 56		280 25.8
	8	347 00 45		323 36.6		7	339 14 03		316 03.0
	11	82 00 38		62 32.4		8	348 28 34		325 16.1
	12	84 24 38		64 58.6		11	82 54 04		59 40.8
	13	87 55 53		68 28.8		12	85 15 19		62 02.3
	19	307 07 34		187 43.6		13	88 42 46		65 30.8
374	4	295 50 18	+21 53.6	273 58.6		19	308 30 54		285 21.4
	5	301 42 25		279 51.7	376	2	293 48 15	+23 33.7	270 15.4
	6	303 07 23		281 15.7		4	296 42 23		273 10.7
	7	338 45 09		316 55.9		5	302 35 28		279 01.1
	8	347 59 04		326 02.9		6	304 01 03		280 28.2
376	7	339°46'55"	+23°33'7	216°10'8	377	12	85°57'01"	+23°33'3	62°21'3
	8	349 03 26		325 28.5		13	89 22 15		65 43.9
	9	13 31 08		389 56.5		19	309 02 38		285 28.3
	11	83 31 16		59 53.1		20	188 59 52		165 25.4
	12	85 51 33		62 17.3		21	221 46 09		198 15.1
	13	89 17 17		65 38.8		22	179 50 46		156 27.4
	16	241 53 26		217 26.1	378	2	294 02 13	+22 56.8	271 01.8
	18	236 27 32		212 58.8		4	296 59 33		274 03.3
	19	308 58 50		285 23.5		5	302 56 13		280 57.5
	20	189 07 32		165 29.4		6	304 22 18		281 26.5
	21	221 52 41		98 21.7		7	340 21 13		317 21.0
	22	179 56 48		156 28.5		8	349 41 17		326 44.3
377	4	296 45 06	+23 33.3	273 13.0		11	84 24 37		61 27.4
	6	304 04 15		280 32.9		12	86 43 40		63 44.2
	7	339 52 03		316 16.4		13	90 06 47		67 12.2
	8	349 09 08		325 35.4		19	309 21 46		286 22.6
	9	13 37 37		350 03.6	379	8	350 18 47	+10 49.4	339 30.8
	11	83 37 00		60 59.6		9	15 45 47		4 58.7

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
379	11	87°59'30"	+10°49'.4	77°08'.9	381	6	300°25'45"	+9°54'.6	290°28'.4
	12	90 15 38		79 17.5		7	335 48 37		325 47.0
	13	98 29 12		82 41.5		8	344 48 33		334 50.1
	14	175 24 18		164 34.1		17	241 13 16		231 21.8
	15	172 57 34		162 09.8		19	304 58 38		294 07.2
	22	180 09 04		169 18.9		21	222 00 32		212 02.6
380	8	350 23 18	+10 01.7	340 20.8		26	265 40 49		255 43.2
	9	15 55 02		5 58.0	382	2	290 58 59	+14 35.7	276 20.1
	11	88 13 33		78 10.1		4	298 59 02		279 28.2
	12	90 29 09		80 23.6		5	299 54 08		285 19.5
	13	98 41 34		83 42.4		6	301 17 12		286 38.8
	14	175 17 44		165 20.1		7	336 40 38		322 00.6
	15	172 51 35		162 47.7		8	345 48 24		331 08.3
	22	180 01 49		170 00.8		19	305 57 49		291 16.0
381	1	267 04 09	+9 54.6	257 15.7	383	2	292 35 49	+20 56.6	271 35.4
	2	290 00 23		280 05.6		4	295 29 49		274 30.6
	4	293 05 23		283 09.9		5	301 21 15		280 21.4
	5	299 03 39		289 06.7		6	302 45 41		281 45.6
	8	347 27 58		326 30.8		26	268 05 38		246 11.3
	19	307 36 43		286 38.3	385	4	296 52 56	+21 41.5	275 07.3
384	1	269 32 18	+21 54.8	247 35.3		5	302 54 46		281 11.9
	2	293 50 49		271 00.5		7	340 35 52		318 52.2
	4	296 52 38		274 02.3		9	14 41 59		35 01.0
	5	302 53 20		280 59.2		11	85 17 39		63 35.7
	6	304 19 25		282 23.6		16	240 57 13		219 13.1
	7	340 28 10		218 30.2		19	309 19 05		287 37.5
	8	349 50 41		227 53.5		20	188 48 34		167 04.4
	9	14 33 01		352 36.0		22	179 56 55		158 15.5
	11	85 06 27		63 08.3	386	1	245 15 19	+20 03.5	225 09.2
	12	87 25 20		65 27.3		4	262 12 37		242 09.1
	13	90 47 17		68 51.8		7	322 08 02		302 03.4
	16	241 08 22		219 12.0		8	328 20 26		308 14.2
	19	309 17 04		287 22.8		17	227 11 40		207 08.7
	20	188 57 16		16 03.2		21	221 46 56		202 42.8
	21	221 17 25		199 20.5		26	241 14 23		221 10.3

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
387	2	293°10'22"	+22°10'.1	270°59'.2	389	4	295°41'51"		272°39'.7
	4	296 02 38		273 51.8		5	302 00 57		288 47.9
	5	301 53 06		279 40.6		6	303 26 54		290 13.9
	6	303 17 59		281 04.5		7	340 23 44		227 09.5
	7	338 51 11		316 37.0		8	349 59 39		336 46.7
	8	348 03 53		325 49.0		11	87 47 56		74 55.4
	19	308 12 23		286 02.9		12	90 05 11		76 48.5
388	1	270 19 45	+23 18.6	24 51.7		13	98 20 32		80 10.0
	2	293 47 13		270 29.8		19	308 15 32		295 01.9
	4	296 39 58		273 18.4		20	189 01 14		175 45.3
	5	302 31 51		279 08.8	390	2	291 16 42	+7 44.5	283 29.8
	6	303 57 27		280 36.7		4	294 48 41		287 05.3
	7	339 40 45		316 22.1		5	301 15 10		293 26.8
	8	348 56 40		325 38.8		6	302 40 28		294 56.3
	16	241 54 42		218 32.1		7	339 53 24		332 13.0
	17	242 28 58		219 02.9		8	349 34 47	+7 44.5	341 48.7
	18	236 28 27		313 11.0		9	15 20 38		7 33.6
	19	308 55 55		285 42.5		11	88 48 41		81 04.4
388	21	221 50 35	+23 18.6	198 28.2		12	91 05 37		83 18.4
	26	268 40 48		245 18.8		13	94 18 29		86 32.4
389	2	292 19 13	+13 12.2	279 04.0		19	307 21 53		299 36.8
						22	189 13 46		181 28.8
391	4	296°19'34"	+14°21'.1	284°00'.4	392	26	265°08'30"	+11°11'.3	253°02'.8
	5	302 36 28		288 15.5	393	2	289 43 24	-1 11.4	290 51.0
	6	304 04 32		289 41.2		4	298 27 09		294 42.5
	19	308 59 23		294 35.4		5	300 02 18		301 16.1
392	2	292 06 25	+11 11.3	280 55.5		6	301 26 26		302 37.4
	4	295 32 38		284 19.9		7	338 56 19		340 04.3
	5	301 55 34		290 46.9		8	348 44 42		349 54.4
	6	303 21 58		292 08.9		9	15 02 04		16 08.4
	7	340 34 07		329 18.1		11	90 02 08		91 13.1
	8	350 14 19		338 57.7		12	92 18 30		98 29.1
	9	15 46 40		4 30.0		13	95 27 50		96 40.3
	11	88 14 21		77 04.2		19	305 56 52		307 08.3
	12	90 30 21		79 23.0		20	189 24 27		190 34.7
	13	93 43 16		82 33.4		22	181 11 41		182 26.0
	18	232 58 50		221 44.5	394	2	289 37 16	-1 37.9	291 14.7
	19	308 11 45		296 57.9		4	298 20 32		294 57.0
	20	188 34 12		177 23.0		7	338 34 30		340 16.5
	21	219 24 35		208 13.2		13	95 30 27		97 07.3
	22	180		169 01.9		22	181 40 13		182 17.0

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
395	4	292°54' 08"	-6°59.7	299°52.8	398	1	260°38' 28"	-23°17.0	283°58.6
	5	299 38 18		306 38.0		2	286 00 53		309 15.9
	7	339 21 38		346 20.7		4	289 51 10		313 10.9
	8	349 23 12		356 23.5		5	296 24 10		319 43.8
	9	15 59 48		23 00.2		6	297 42 56		320 01.9
	21	217 10 02		224 10.0		7	334 01 37		357 17.9
396	4	291 36 20	-13 41.2	305 19.3		8	343 32 05		6 50.7
	5	298 20 19		311 58.0		11	89 19 01		112 40.3
	6	299 43 13		313 25.0		12	91 43 18		114 59.8
	7	337 32 42		351 12.3		13	94 59 55		118 15.1
	8	347 30 12		1 09.4		19	301 41 04		324 58.8
	11	91 00 45		104 44.2		21	218 46 55		242 00.9
	12	98 15 35		107 00.2		26	260 17 11		283 37.1
	13	96 19 49		110 01.4	399	1	261 42 42	-20 39.8	282 19.5
	19	304 02 11		317 42.0		2	286 25 21		307 07.0
397	9	7 01 12	-21 21.3	28 25.1		4	290 06 20		310 49.7
	11	88 14 35		109 36.5		7	333 40 10		354 28.8
	12	90 50 27		112 08.9		8	342 58 34		3 40.6
	13	94 23 22		115 46.4		9	8 45 11		9 27.0
399	11	87°17' 47"	-20°39.8	107°58.7	401	2	286°51' 56"	-17°19.2	304°13.7
	12	89 46 30		110 25.3		4	290 25 32		307 41.2
	13	98 11 49		113 50.5		5	296 44 27		314 05.3
	18	232 07 48		252 49.2		7	333 31 30		350 48.8
	20	190 56 11		211 34.5		8	342 40 50		359 59.7
	21	219 26 57		240 11.5		9	7 58 14		25 17.5
	22	182 17 15		202 59.3		11	85 43 37		103 01.8
	26	261 07 57		281 48.0		12	88 15 56		105 37.4
400	2	286 59 48	-17 40.7	304 44.4		13	91 48 32		109 09.4
	4	290 32 51		308 14.1		19	302 07 35		319 29.0
	7	333 48 36		351 29.9		20	191 34 03		208 52.3
	8	343 01 04		0 36.2		21	220 13 21		237 38.0
	9	8 22 07		26 01.6		22	182 43 15		200 04.8
	11	85 40 42		103 18.0		26	261 57 46		279 19.0
	12	88 11 26		105 49.6	402	2	287 55 16	-7 42.4	295 37.0
	13	91 42 08		109 26.4		4	291 14 59		299 01.3
	21	219 58 26		237 41.9		5	297 22 52		305 04.1
	22	182 18 34		199 59.8		6	298 42 20		306 28.1
401	1	262 43 14	-17 19.2	280 20.0		7	333 44 49		341 23.3

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
402	8	342°42'56"	-7°42'4	350°23'0	403	13	87°17'54"	+15°20'8	71°55'1
	9	7 16 24		14 59.7		19	305 58 42		290 37.7
	10	8 04 21		15 46.1		20	192 26 50		177 01.2
	11	82 53 32		90 32.7		25	340 19 46		324 55.8
	12	85 30 03		93 12.8	404	2	291 10 57	+15 27.6	275 45.6
	13	89 13 24		96 53.2		4	294 08 58		278 44.1
	19	302 54 28		310 35.5		5	300 01 12		284 36.9
	20	192 37 42		200 20.3		7	336 28 43		321 04.8
	22	183 24 22		191 08.0		8	345 29 41		330 07.1
403	2	291 10 13	+15 20.8	275 50.5		9	9 39 41		354 14.4
	4	294 08 16		278 47.5		10	10 33 49		355 02.6
	5	300 00 34		284 44.0		11	81 05 56		65 35.6
	6	301 22 45		285 03.0		12	83 36 35		68 07.4
	7	336 28 12		321 10.4		13	87 17 52		71 52.5
	8	346 29 12		331 10.3		19	305 59 21		290 36.6
	9	9 39 19		354 13.8		20	192 27 42		177 00.1
	10	10 33 29		355 15.8	405	1	264 54 32	-7 57.7	272 53.3
	11	81 06 01		65 46.5		2	287 58 10		295 57.0
	12	83 36 40		68 16.8		4	291 17 00		299 14.1
405	5	297°24'00"	-7°57'7	305°21'6	406	8	343°07'03"	-2°27'4	345°33'0
	6	298 43 20		306 42.7		9	7 25 44		9 53.8
	7	333 40 46		341 38.4		10	8 12 41		10 43.2
	8	342 36 46		350 33.6		11	81 45 35		84 12.8
	9	7 04 56		15 03.8		12	84 22 20		86 51.2
	14	179 29 54		187 28.8		13	88 08 26		90 33.7
	15	177 09 54		185 10.0		14	179 18 24		181 45.3
	16	239 51 24		247 52.0		15	176 55 12		179 19.3
	18	235 05 55		243 05.9		16	240 32 38		243 04.6
	19	302 54 54		310 48.4		17	240 58 48		243 27.7
	21	221 58 10		229 57.3		19	303 32 01		306 02.2
	22	183 41 03		191 40.1		21	222 26 57		224 54.5
	26	263 48 11		271 47.7		22	183 57 48		186 04.3
406	1	265 51 50	-2 27.4	268 19.2	407	1	268 25 45	+13 29.5	254 57.0
	2	288 38 56		291 07.9		2	290 49 43		277 22.3
	4	291 51 42		294 21.2		4	293 49 27		280 20.6
	5	297 54 19		300 19.3		5	299 42 49		286 14.3
	6	299 14 20		301 39.5		6	301 04 41		287 38.1
	7	334 11 16		336 38.3		7	336 07 51		322 39.1

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}	
407	8	345 ⁰ 07'36"	+13 ⁰ 29'.5	331 ⁰ 36'.8	408	8	345 ⁰ 54'21"	+17 ⁰ 25'.4	328 ⁰ 25'.9	/270
	9	9 17 19		355 50.4		9	10 06 18		352 39.2	
	10	10 09 55		356 41.0		10	11 01 55		353 34.1	
	11	81 03 17		67 36.9		11	81 25 32		64 01.1	
	12	83 35 04		70 04.6		12	83 55 17		66 26.2	
	13	87 17 38		73 44.2		13	87 35 12		70 06.7	
	16	242 14 21		228 49.5		16	242 32 40		225 07.0	
	17	242 36 57		229 02.3		17	242 56 16		225 27.8	
	19	305 37 59		292 6.8		19	306 21 09		288 51.2	
	20	192 40 44		179 08.4		20	192 21 39		174 58.2	
	21	223 28 57		209 59.7		21	223 34 11		206 08.2	
	22	183 02 40		169 28.1		22	182 44 26		165 17.5	
	26	266 55 12		253 28.6		26	267 30 20		250 04.7	
408	1	269 03 22	+17 25.4	251 37.7	409	1	269 33 23	+20 37.4	248 58.6	
	2	291 31 22		274 06.9		2	292 12 27		271 35.8	
	4	294 28 59		277 06.2		4	295 09 08		274 33.0	
	5	300 21 10		282 57.8		5	301 01 12		280 25.7	
	6	301 43 35		284 20.2		6	302 24 14		281 47.5	
	7	336 52 01		319 27.2		7	337 40 19		317 04.3	
409	8	346 ⁰ 45' 53"	+20 ⁰ 37'.4	326 ⁰ 08'.0	411	5	301 ⁰ 12'05"	+22 ⁰ 01'.0	279 ⁰ 12'.6	/271
	9	11 02 12		350 20.7		6	302 35 02		280 34.2	
	10	12 01 26		351 27.1		7	337 51 47		315 52.5	
	11	82 03 59		61 28.7		8	346 57 24		324 54.5	
	12	84 31 33		63 52.4		9	11 14 58		349 16.3	
	13	88 08 15		67 29.5		10	12 14 00		350 11.7	
	17	243 07 13		222 33.1		11	82 26 10		60 29.6	
	19	307 05 51		286 28.2		12	84 53 57		62 51.6	
	20	191 55 55		171 26.6		13	88 30 44		666 28.6	
	21	223 30 12		202 52.1		16	242 51 26		220 51.0	
	22	182 22 46		161 47.1		17	243 17 24		221 17.8	
	26	267 59 21		247 22.1		18	237 47 16		215 47.0	
410	2	288 19 52	-3 47.1	292 08.5		19	307 15 42		285 17.2	
	4	291 33 54		295 22.9		20	192 14 49		180 11.0	
	6	298 56 04		302 44.7		21	223 43 06		201 43.7	
	21	222 50 58		226 40.2		22	182 42 51		160 46.0	
411	1	269 39 39	+22 01.0	247 37.1		23	179 42 22		157 37.4	
	2	292 21 28		270 21.0		26	268 06 59		246 07.1	
	4	295 19 12		273 15.4	412	2	292 51 00	+23 17.1	269 33.6	

TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	χ	P _{obs}	No.	No. of cra- ter	P _{comp}	χ	P _{obs}
412	4	295°50'55"	+23°17'.1	272°32'.1	413	10	17°53'58"	+16°18'.1	1°33'.7
	5	301 46 45		278 29.0		11	88 22 45		72 05.2
	6	303 10 29		279 52.3		12	90 36 58		74 17.5
	7	338 43 35		315 27.1		13	93 47 22		77 29.4
	8	347 54 55		324 39.4	414	1	268 08 09	-15 02.5	283 11.3
	9	12 23 20		349 07.9		2	290 12 03		305 15.0
	10	13 27 56		350 11.9		4	298 13 23		308 16.1
	11	83 34 41		60 19.9		5	299 07 01		314 12.3
	12	85 59 37		62 39.4		6	300 27 46		315 28.0
	13	89 31 21		66 15.4		16	242 33 10		257 34.2
	16	242 25 44		219 05.7		17	242 51 58		257 55.2
	17	242 56 29		219 36.8		18	237 46 58		252 48.3
	18	237 14 35		213 59.3		19	304 53 38		320 00.9
	19	307 55 08		284 40.5		26	266 38 52		281 37.3
	21	223 06 16		199 50.4	415	1	269 28 52	+21 12.9	248 14.6
	22	182 03 48		158 45.1		2	291 59 51		270 42.6
	23	179 06 16		155 50.1		4	294 58 11		273 45.0
413	8	350 51 37	+16 18.1	334 31.5		5	300 50 57		279 34.1
	9	10 24 43		354 02.8		6	302 13 22		281 00.5
415	7	337°22'50"	+21°12'.1	316°05'.8	416	21	222°54'22"	+23°28'.7	199°20'.3
	8	346 25 27		325 12.2		26	267 54 10		244 21.7
	18	237 59 18		216 50.7	417	2	290 40 00	+10 05.5	280 34.0
	19	306 50 32		285 37.3		4	294 19 00		284 08.0
	20	192 51 37		171 41.0		5	300 52 36		290 50.4
	21	224 01 35		202 48.1		6	302 18 32		292 15.4
	22	183 15 41		162 03.9		7	340 00 00		329 53.8
	23	180 14 19		169 04.2		8	349 49 45		339 45.6
	26	267 56 46		216 42.9		9	15 53 31		5 52.5
416	2	292 45 30	+23 28.7	269 18.4		10	17 21 55		7 15.0
	4	295 49 31		272 20.0		11	89 30 49		79 22.5
	5	301 48 09		278 20.6		12	91 45 30		81 41.0
	6	303 11 54		279 41.1		13	94 53 56		84 46.5
	7	338 51 43		315 20.3	418	1	268 58 19	+4 50.5	264 06.8
	8	348 04 59		324 35.1		2	292 45 26		287 54.4
	16	242 09 09		218 42.6		4	295 53 06		291 06.6
	17	242 41 56		219 15.2		5	301 55 46		297 07.3
	19	307 55 27		284 24.3		6	303 19 29		298 31.8
	20	191 22 23		167 53.3		7	339 08 43		334 20.4

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TABLE 3, Continued

No.	No. of cra- ter	P _{comp}	X	P _{obs}	No.	No. of cra- ter	P _{comp}	X	P _{obs}
418	8	349°24'43"	+4°50'5	343°31'5	420	4	294°29'00"	+11°41'2	282°49'9
	16	241 54 20		237 02.7		5	300 59 14		289 19.9
	17	242 29 50		237 38.2		7	339 50 45		328 08.4
	19	308 01 01		303 10.2		8	349 36 10		337 59.8
	21	222 47 36		217 52.9		9	15 32 21		3 52.5
	22	182 25 21		177 34.9		10	16 57 52		5 21.4
	26	267 39 12		262 48.8		11	89 13 44		77 32.7
419	4	295 33 26	+18 58.7	276 37.5		12	91 29 43		79 48.8
	5	301 50 03		282 50.1		13	94 40 27		83 03.3
	6	303 15 01		28 13.2		16	237 36 49		225 59.4
	7	339 55 10		321 56.4		19	307 05 18		295 22.8
	8	349 26 35		330 32.4		20	188 56 42		177 11.1
	10	16 04 18		357 10.0		21	219 05 35		207 28.3
	16	239 42 25		220 43.7		22	180 41 20		168 57.0
	17	240 30 02		221 31.3		26	263 50 09		252 09.8
	18	234 23 29		215 25.2	421	4	293 39 34	+6 23.9	287 17.7
	19	307 58 34		288 02.0		5	300 17 11		293 51.7
	22	181 18 54		162 17.2		7	339 33 27		333 09.9
420	2	290 52 59	+11 41.2	279 08.9		8	349 26 04		343 02.3
421	9	15°45'19"	+6°23'9	5°22'7					
	10	17 14 29		10 52.9					
	11	89 59 12		83 39.6					
	13	95 20 12		88 57.7					
	19	306 20 17		299 54.7					
	20	188 28 13		182 03.8					
	22	180 22 24		173 57.3					

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